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THE WERNER ARITHMETIC

FRANK H. HALL

BOOK III

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THE WERNER
ARITHMETIC

ORAL AND WRITTEN

BOOK THREE—PARTS I AND II

BY
FRANK H. HALL
AUTHOR OF "ARITHMETIC READERS," AND "ARITHMETIC OF FARM AND WORKSHOP"

CHICAGO NEW YORK
WERNER SCHOOL BOOK COMPANY

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PREFACE.

In Books I and II of this series, classification is made subordinate to gradation. In Book III, it is assumed that the pupil has a body of knowledge sufficient to enable him to profit by a topical arrangement of subjects. Classification and generalization are therefore the features of this book.

In answer to what is believed to be a general demand, the number of topics has been greatly reduced. The fundamental operations in their application to simple numbers, decimals, United States money, denominate numbers, and literal quantities, are treated briefly under the four general heads, *Addition*, *Subtraction*, *Multiplication*, and *Division*. The other topics presented are,—*Properties of Numbers*, *Divisibility of Numbers*, *Fractions*, *Percentage and Its Applications*, *Ratio and Proportion*, *Powers and Roots*, *The Metric System*, and *Denominate Numbers*.

Each page of the book is a unit of the greater ten-page unit. Upon the first *six pages* of every *ten pages*, a part or the whole of some general topic is presented. Upon the *seventh and eighth pages*, the algebraic phase of this topic appears. The *ninth page* is devoted to elementary work in geometry, and the *tenth* to miscellaneous problems. This arrangement makes the book convenient for reference and review, and will aid the pupil in gaining a perception of the relation of the topics and sub-topics.

The introduction of the elements of algebra and geometry will greatly increase the interest of all the pupils, will prove invaluable to those who leave school at the end of the eighth year, and become a helpful stepping stone to such as take a high school course.

The author is under great obligation to Supt. J. C. Burns of Monmouth, Illinois, whose criticism of the manuscript of Book III has greatly improved the form of expression, and to Dr. J. B. Shaw, Professor of Mathematics in Illinois College, to whom many queries have been submitted, the answers to which have helped to bring the book into harmony with the best mathematical thought of the day. In justice to these scholars, it should be said that neither of them is responsible for any errors that may be found in the book.

For all that is new and peculiar on pages 211 and 213, the author is indebted to Mr. Geo. R. Parker, a blind man who has taught mathematics in The Illinois Institution for the Education of the Blind with marked success for many years.

F. H. H.

JACKSONVILLE, ILLINOIS, June, 1898.

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INTRODUCTION.

NOTE.—The following brief statements concerning the elementary branches of mathematics, and the place of arithmetic in mathematical science, are designed for the teacher rather than for the pupil. After the completion of the book, these statements should be read aloud in class and the pupil encouraged to comment upon them and to discuss their meaning. After this has been done, the pupils may be required to commit to memory such parts as seem most important.

1. Mathematics treats of the measurement and comparison of magnitudes.

Geometry, arithmetic, and algebra are the elementary branches of mathematics.

2. Geometry treats of the measurement and comparison of certain magnitudes known as lines, angles, surfaces, and solids.

Measurement makes number necessary.

Primarily, number signifies ratio. The number six suggests that some magnitude, A, is six times some other magnitude, B; and, conversely, that the magnitude, B, is one sixth of the magnitude, A.

Number *is* ratio.

Secondarily, number signifies aggregation. The number six suggests the aggregation of six minor magnitudes into one group, or major magnitude.

NOTE.—It is the secondary aspect of number that the child first apprehends. Thus, that which is logically secondary is pedagogically primary. To the child, the number six suggests a group of six (marbles, apples, inches). It is undoubtedly true that the mature mind ordinarily conceives of number in its aggregative aspect. It is the mathematician only who habitually thinks of number as *ratio*.

3. Arithmetic and Algebra treat of measured magnitudes and their relation—hence of number.

Every arithmetical or algebraic problem is one of the following two simple problems, or may be resolved into such as these:

I. Given, two measured magnitudes, to find their relation.

II. Given, one measured magnitude and its relation to another magnitude, to find the measurement of the second magnitude.

Relation may be (1) by difference or (2) by quotient.

4. Every branch of mathematics is concerned with number and its expression; but it is the special province of arithmetic (1) to present the common method of numerical notation, (2) to investigate the properties and laws of number, and (3) to teach the art of computation in the simple processes of reduction, addition, subtraction, multiplication, and division.

NOTE.—For the double purpose of giving practice in simple computations and of preparing pupils for the ordinary number work of life, applications of these processes to certain classes of business transactions are presented in all arithmetical text-books.

5. It is the province of Algebra, by means of literal and figure notation combined, (1) to abridge and (2) to generalize the various processes of computation.

6. A unit is one.

7. A unit of measurement is a standard of measurement; as, a foot, a quart, an hour, a dollar, etc.

NOTE.—Counting is measuring, though sometimes inexact, owing to lack of uniformity in the magnitudes counted.

8. A unit of number is one of number.

NOTE 1.—A unit of number is, in a sense, a unit of measurement. It is a standard for measuring number.

NOTE 2.—The unit of number may be simply, one or one ten, or one hundred, or one tenth, or one thousandth; thus, we may have units of the first order, units of the second order, units of the first decimal order, etc. We may have, too, fractional units other than decimal; as, one fourth, one fifth, one twenty-first, etc. A unit is a one, or any

group regarded for the time as one. Any unit, however great, may become a part of a major unit, and any unit, however small, may be regarded as made up of minor units.

NOTE 3.—It is sometimes said that a unit is a standard by which we count or measure, and that a number is a unit or a collection of units. If this be true, *six boys*, *six marbles*, *six books*, etc., are numbers!

9. A number that is joined to the name of a standard of measurement is said to be a *concrete number*.

NOTE.—Neither *four feet*, *six hours*, nor *eight apples* is a concrete number; the *four*, *six*, and *eight* of these expressions are concrete numbers. If eight apples is a concrete number, then a red apple is a concrete color!

10. A number that is not joined to the name of a standard of measurement, as *four*, *five*, *eight*, *ten*, is said to be an abstract number.

NOTE 1.—An abstract number may *suggest* simply a ratio, or it may suggest both a ratio and a measured magnitude. In the expression, 8 feet ÷ 2 feet = 4, the four suggests pure number, *i.e.*, simply a ratio —the ratio of the magnitude 8 feet to the magnitude 2 feet. The expression $6 + 5 = 11$, *may* suggest that 6 inches increased by 5 inches is equal to 11 inches. One may also think the 6 or the 5 or the 11 as the ratio of a certain magnitude to the standard of measurement.

NOTE 2.—Strictly speaking, number is ratio and, from its very nature, is always abstract. But the expressions, "*concrete number*," "*denominate number*," etc., have been accorded a place in the language. They are convenient terms to express *certain uses of abstract number* and hence are freely employed on the pages of this book.

ORDER OF PROCEDURE IN TRAINING FOR ARITHMETICAL POWER.

STEP I.

Since arithmetic is concerned mainly with the comparison of measured magnitudes and their numerical expression, it is of prime importance that the work should *begin in the actual comparison and measurement of sentient objects.*

STEP II.

Since by far the greater part of the actual work in arithmetic must be done without the presence of the sense magnitudes compared, it is equally important that the pupil should early learn to *image measured magnitude and to compare the images of measured magnitudes.*

NOTE.—The imaging of magnitude, as the words are here used, means the mental reproduction of that which has been in the mind during an act of sense perception with special attention to quantity.

STEP III.

Since most of the magnitudes compared by the mathematician never have been to him objects of sense perception, and some of them never can be, it is no less important that the pupil should be constantly trained in *the creation of imaginary measured magnitudes.*

NOTE 1.—A most serious defect in ordinary arithmetical training is, that pupils *image figures* instead of magnitudes—that they are expected to see relation when the magnitudes compared are not present in consciousness.

NOTE 2.—The multiplication of sense-experiences in the comparison of magnitudes will not alone give the desired power. To learn to work with ideal magnitudes, one must *work with ideal magnitudes.* "Object teaching" must be vigorously begun and as vigorously laid aside.

NOTE 3.—In the lower grades, great emphasis must be put upon the actual measurement of sense-magnitudes. In the higher grades, the chief part of the work will be in the comparison of imaginary magnitudes. But there is no grade from which Step I. can be entirely omitted, and no grade (in which formal number work should be taught) where Step I. should occupy as much time as Step II., or where Step II. should occupy as much time as Step III.

THE WERNER ARITHMETIC.

BOOK THREE—PART ONE.

NOTATION.

1. The expression of numbers by symbols is called **notation.**

2. In mathematics two sets of symbols are employed to represent numbers; namely, ten characters—1, 2, 3, 4, **5**, 6, 7, 8, 9, 0—called **figures**, and the **letters**, *a*, *b*, *c*, *d*, . . . *x*, *y*, *z*.

NOTE.—The figures from 1 to 9 are called *digits*. The term *significant figures* is sometimes applied to the digits. The tenth character (0) is called a cipher, zero, or naught.

The Arabic Notation.

3. The method of representing numbers by figures and places is called the **Arabic Notation**. It is the *principle of position* in writing numbers that gives to the system its great value.

2	4	3	8.	5	9	6	2	4	3	8.5 9 6
Fourth place.	Third place.	Second place.	First place.	First decimal place.	Second decimal place.	Third decimal place.	Units of the fourth order.	Units of the third order.	Units of the second order.	**Primary units.** Units of the first decimal order. Units of the second decimal order. Units of the third decimal order.

4. A figure standing alone or in the first place represents primary units, or units of the first order; a figure standing in the second place represents units of the second order; a figure standing in the third place represents units of the third order; a figure standing in the first decimal place represents units of the first decimal order, etc.

5. The following are the names of the units of eight orders:

Fourth decimal order . . .	ten-thousandths.
Third decimal order . . .	thousandths.
Second decimal order . . .	hundredths.
First decimal order	tenths.

DECIMAL POINT.

First order	primary units.
Second order	tens.
Third order	hundreds.
Fourth order	thousands.

6. What are the names of the units represented by each figure in the following? 3265.8419.

7. In a row of figures representing a number, the figure on the right represents the lowest order given; the figure on the left, the highest order given. In general, any figure represents an order of units higher than the figure on its right (if there be one), and lower than the figure on its left (if there be one).

8. Ten units of any order equal one unit of the next higher order; thus, ten hundredths equal one tenth; ten tenths equal one primary unit, etc.

9. The naught, or zero, is used to mark vacant places; thus, the figures 205 represent 2 hundred, no tens, and 5 primary units.

NOTE 1.– Observe that a figure *always stands for units.* If it occupies the first place, it stands for primary units; if it occupies the second place, it stands for tens (that is, units of tens); the third place, for hundreds; the first decimal place, for tenths; the second decimal place, for hundredths, etc. Thus, a figure 5 always stands for five—five primary units, five thousand, five hundredths, five tenths, according to the place it occupies.

NOTE 2.—In reading integral numbers the primary unit should be, and usually is, most prominent in consciousness. Thus, the number 275 is made up of 2 hundreds, 7 tens, and 5 primary units; but 2 hundreds equal two hundred (200) primary units, and seven tens equal seventy (70) primary units; these $(200 + 70 + 5)$ we almost unconsciously combine in our thought, and that which is ordinarily present in consciousness is 275 *primary units.* So in the number 125,246, there are units of six orders, which we reduce in thought to primary units and say, one hundred twenty-five thousand two hundred forty-six *primary units.*

NOTE 3.—In reading decimals, too, the primary unit should be prominent in consciousness. Thus, .256 is made up of 2 tenths, 5 hundredths, and 6 thousandths; but 2 tenths equal 200 thousandths, and 5 hundredths equal 50 thousandths; these $(200 + 50 + 6)$ we combine in our thought, and that which should be present in consciousness is 256 thousandths *of a primary unit.*

10. EXERCISE.

Write in figures:

1. Two hundred fifty-four thousand one hundred sixty.
2. One hundred seventy-five and two hundred six thousandths.
3. Eighty-four and three hundred twenty-five thousandths.
4. One hundred ninety-seven and twenty-seven hundredths.
5. Seven thousand four hundred twenty-four and six tenths.
6. Twenty-four thousand six hundred fifty-one.
7. One hundred thirty-five thousand two hundred fifty.

(a) *Find the sum of the seven numbers.*

11. Exercise.

Read in two ways as suggested in the following:

324.61. (1) 3 hundreds, 2 tens, 4 primary units, 6 tenths, 1 hundredth. (2) Three hundred twenty-four and sixty-one hundredths.

1. 2746.2
2. 546.85
3. 24.006
4. 1.6285
5. 286.35
6. 2651.4
7. 80.062
8. 2085.7
9. 120.08
10. 32.405

Use the word *and* in place of the decimal point only.

12. Exercise.

Observe that any number may be read by giving the name of the units denoted by the right-hand figure, to the entire number: thus, 146 is 146 primary units; 21.8 is 218 tenths; 3.25 is 325 hundredths.

1. 27 = 2 tens + 7 primary units = 27 primary units.
2. 2.7 = 2 primary units + 7 tenths = ——— tenths.
3. .27 = 2 tenths + 7 hundredths = ——— hundredths.
4. .027 = 2 hundredths + 7 thousandths = ——— thousandths.
5. .436 = 4 tenths + 3 hundredths + 6 thousandths = ——— thousandths.
6. 5.247 = 5 primary units + 2 tenths + 4 hundredths + 7 thousandths = 5247 ———ths.
7. 3.24 = ——— hundredths.
8. 5.206 = ——— thousandths.
9. 25.13 = ——— hundredths.
10. 14.157 = ——— thousandths.
11. 275.4 = ——— tenths.
12. 275.12 = ——— hundredths.

Note.—Exercise 12 and Exercise 13 are important as a preparation for the clear understanding of division of decimals. Do not omit them nor permit the work to be done carelessly.

13. Exercise.

Observe that any part of a number may be read by giving the name of the units denoted by the last figure of the part, to the entire part; thus, 24.65 is 246 tenths and 5 hundredths; 14.275 is 1427 hundredths and 5 thousandths. In a similar manner read each of the following:

1. 2.75 = ——— tenths and ——— hundredths.
2. 32.46 = ——— tenths and ——— hundredths.
3. 1.425 = ——— hundredths and ——— thousandths.
4. 24.596 = ——— tenths and ——— thousandths.
5. 321.45 = ——— tenths and ——— hundredths.
6. 14.627 = ——— hundredths and ——— thousandths.
7. 2.6548 = ——— hundredths and ——— ten-thousandths.
8. 32.479 = ——— tenths and ——— thousandths.
9. 21467 = ——— hundreds and ——— primary units.
10. 16853 = ——— tens and ——— primary units.

14. Exercise.

Observe that in reading a mixed decimal in the usual way, we divide it into two parts and give the name of the units denoted by the last figure of each part to each part; thus, 2346.158 is read 2346 (primary units) and 158 thousandths.

Read the following in the usual manner. Do no not use the word *and* in reading the numbers in the second column:

1. 200.006	.206	6. 800 and 24.	824.
2. 400.0005	.0405	7. 9000 and 6.	9006.
3. 500.025	.525	8. 2400 and 8.	2408.
4. 200 and 40.	240.	9. 17000 and 4.	17004.
5. 700 and 35.	735.	10. 46500 and 40.	46540.

15. Exercise.

Write in figures:

1. Two hundred and eight thousandths.
2. Two hundred eight thousandths.
3. Six hundred and twelve thousandths.
4. Six hundred twelve thousandths.

Reference Table.

Tredecillions.	Duodecillions.	Undecillions.	Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Primary units.
275,	301,	246,	157,	896,	275,	832,	456,	297,	143,	215,	367,	291,	326.	415.

NOTE 1.—The names of the periods above tredecillions are: quatuordecillion, quindecillion, sexdecillion, septendecillion, octodecillion, novemdecillion and vigintillion.

16. Note the number of decimal places in each of the following expressions:

1. .4 = 4 tenths. (1 decimal place.)
2. .27 = 27 hundredths. (2 decimal places.)
3. .346 = 346 thousandths. (3 decimal places.)
4. .2758 = 2758 ten-thousandths.
5. .07286 = 7286 hundred thousandths.
6. .000896 = 896 millionths. (6 decimal places.)
7. .000,468,275 = ——— billionths. (9 decimal places.)
8. .000,000,000,462 = ——— trillionths.
9. .000,000,000,000,527 = ——— quadrillionths.
10. .000,000,000,000,000,004 = ——— quintillionths.
11. .000,000,000,000,000,000,037 = ——— ———.
12. .000,000,000,000,000,346,275 = ——— ———.
13. .000,000,000,000,002,427,836 = ——— ———.
14. In any number of thousandths there are ——— decimal places.
15. In any number of millionths there are ——— decimal places.
16. In any number of billionths there are ——— decimal places.

Algebra—Notation

17. Letters are used to represent numbers; thus, the letter a, b, or c may represent a number to which any value may be given.

18. Known numbers, or those that may be known without solving a problem, when not expressed by figures, are usually represented by the first letters of the alphabet; as, a, b, c, d.

ILLUSTRATIONS.

(a) To find the perimeter of a square when its side is given.

Let a = one side.*

Then $4a$ = the perimeter.

Hence the rule: *To find the perimeter of a square, multiply the number denoting the length of its side by 4.*

(b) To find the perimeter of an oblong when its length and breadth are given.

Let a = the length.

Let b = the breadth.

Then $2a + 2b$, or $(a + b) \times 2$ = the perimeter.

Hence the rule: *To find the perimeter of an oblong, multiply the sum of the numbers denoting its length and breadth by 2.*

19. Unknown numbers, or those which are to be found by the solution of a problem, are usually represented by the last letters of the alphabet; as, x, y, z.

ILLUSTRATION.

(a) There are two numbers whose sum is 48, and the second is three times the first. What are the numbers?

Let x = the first number.

Then $3x$ = the second number,

and $x + 3x = 48$.

$4x = 48$.

$x = 12$.

$3x = 36$.

The numbers are 12 and 36.

* That is, the *number* of units in one side. The letter stands for the number.

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Tredecillions.	Duodecillions.	Undecillions.	Decillions.	Nonillions.	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Primary units.
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8. A unit of number is one of number.

NOTE 1.—A unit of number is, in a sense, a unit of measurement. It is a standard for measuring number.

NOTE 2.—The unit of number may be simply, one or one ten, or one hundred, or one tenth, or one thousandth; thus, we may have units of the first order, units of the second order, units of the first decimal order, etc. We may have, too, fractional units other than decimal; as, one fourth, one fifth, one twenty-first, etc. A unit is a one, or any

group regarded for the time as one. Any unit, however great, may become a part of a major unit, and any unit, however small, may be regarded as made up of minor units.

NOTE 3.—It is sometimes said that a unit is a standard by which we count or measure, and that a number is a unit or a collection of units. If this be true, *six boys*, *six marbles*, *six books*, etc., are numbers!

9. A number that is joined to the name of a standard of measurement is said to be a *concrete number*.

NOTE.—Neither *four feet*, *six hours*, nor *eight apples* is a concrete number; the *four*, *six*, and *eight* of these expressions are concrete numbers. If eight apples is a concrete number, then a red apple is a concrete color!

10. A number that is not joined to the name of a standard of measurement, as *four*, *five*, *eight*, *ten*, is said to be an abstract number.

NOTE 1.—An abstract number may *suggest* simply a ratio, or it may suggest both a ratio and a measured magnitude. In the expression, 8 feet + 2 feet = 4, the four suggests pure number, *i.e.*, simply a ratio —the ratio of the magnitude 8 feet to the magnitude 2 feet. The expression $6 + 5 = 11$, *may* suggest that 6 inches increased by 5 inches is equal to 11 inches. One may also think the 6 or the 5 or the 11 as the ratio of a certain magnitude to the standard of measurement.

NOTE 2.—Strictly speaking, number is ratio and, from its very nature, is always abstract. But the expressions, "*concrete number*," "*denominate number*," etc., have been accorded a place in the language. They are convenient terms to express *certain uses of abstract number* and hence are freely employed on the pages of this book.

36. EXAMPLES OF ADDITION

1.	2.	3.
3754	37.426	$24.305
2862	1.48	$6.752
1457	375.062	$375.08
8073	413.968	$406.137

4.	5.	6.
275 acres.	43 gal. 2 qt. 1 pt.	$5a + 2b$
146 acres.	24 gal. 1 qt. 1 pt.	$27a + 3b$
27 acres.	63 gal. 3 qt. 1 pt.	$46a + 4b$
448 acres.	131 gal. 3 qt. 1 pt.	$78a + 9b$

7. In each of the above examples, tell which are addends and which is the sum.

37. *Observe* that in written problems in addition the figures that stand for units of the same order are usually written in the same column.

1. In example 2, what figures represent units of the second decimal order? Of the third decimal order?

2. In example 5, what figures represent units of the first integral order?

38. *Observe* that in written problems in addition of denominate numbers, the figures that stand for units of the same denomination and order are usually written in the same column.

1. In example 5, what figures represent units of gallons? of quarts?

2. In example 5, what figures represent tens of gallons?

3. In example 4, what figures represent hundreds of acres?

Addition—Simple Numbers.

39. Find the sum of 275, 436, and 821.

Operation.

$$\begin{array}{r} 275 \\ 436 \\ 821 \\ \hline 1532. \end{array}$$

Explanation.

The sum of the units of the first order is 12; this is equal to one unit of the second order and 2 units of the first order. Write the 2 units of the first order, and add the 1 unit of the second order to the other units of the second order.

The sum of the units of the second order is 13; this is equal to 1 unit of the third order and 3 units of the second order. Write the 3 units of the second order, and add the 1 unit of the third order to the other units of the third order.

The sum of the units of the third order is 15; this is equal to 1 unit of the fourth order and 5 units of the third order, each of which is written in its place.

The sum of 275, 436, and 821 is 1532.

40. PROBLEMS.

1. Add 3465, 4268, 3279, 6534, 5731.
2. Add 5732, 6721, 3466, 4269, 6535.
3. Add 2768, 5329, 4685, 3752, 8467.
4. Add 4671, 5315, 6248, 1533, 7232.

5.	6.	7.	8.
375	436	625	564
506	307	494	693
258	316	742	684
327	498	673	502
580	736	420	264
346	274	654	726
846	123	154	877
362	888	638	112

(*a*) Find the sum of the eight sums.

TO THE TEACHER.—Impress upon the pupil the fact that in arithmetic nothing short of accuracy is commendable. One figure wrong in one problem in ten is failure. The young man or the young woman who cannot solve ten problems like those on this page, without an error, is worthless as an accountant.

Addition—Decimals.

41. Find the sum of 4.327, 8.29 and .836.

Operation.	*Explanation.*
4.327 8.29 .836 13.453	The sum of the units of the third decimal order is 13; this is equal to 1 unit of the second decimal order and 3 units of the third decimal order. Write the 3 units of the third decimal order and add the 1 unit of the second decimal order to the other units of that order.

The sum of the units of the second decimal order is 15; this is equal to 1 unit of the first decimal order and 5 units of the second decimal order. Write the 5 units of the second decimal order and add the 1 unit of the first decimal order to the other units of that order.

The sum of the units of the first decimal order is 14; this is equal to 1 unit of the first integral order and 4 units of the first decimal order. Write the 4 units of the first decimal order and add the 1 unit of the first integral order to the other units of that order.

The sum of the units of the first integral order is 13; this is equal to 1 unit of the second integral order and 3 units of the first integral order, each of which is written in its place.

The sum of 4.327, 8.29, and .836 is 13.453.

42. Problems.

1. Add 274.36, 21.37, 38.007, and .275
2. Add 78.63, 61.993, .725, and 724.64
3. Add .7, .84, .375, .0275, and .25326
4. Add .16, .625, .9725, .74674, and .3
5. Add 46.07, 14.003, 52.0006, and 28
6. Add 85.997, 47.9994, 72, and 53.93
7. Find the sum of two hundred and six thousandths, and two hundred six thousandths.
8. Find the sum of seven hundred ninety-eight and nine hundred ninety-four thousandths, and seven hundred ninety-four thousandths.

(*a*) Find the sum of the eight sums.

To the Pupil. – Can you solve these eight problems and find the sum of the eight sums on first trial without an error?

Addition—United States Money.

43. Find the sum of the money represented in the following columns:

$324.45
28.47
375.28
6.94
175.89
27.56
475.39
802.21
354.48
916.37
144.50
75.34
8.88
246.25
$3962.01

Explanation.

Sums of the units of each order:

Second decimal order (cents)	81
First decimal order (dimes)	70
First integral order	82
Second integral order	56
Third integral order	39

Observe that the 8 of the first sum is included in the 70 of the second sum; that the 7 of the second sum is included in the 82 of the third sum; that the 8 of the third sum is included in the 56 of the fourth sum, and that the 5 of the fourth sum is included in the 39 of the fifth sum. Hence, the sum of the five sums will be represented by the figures 396201.

44. PROBLEMS.*

1.	2.	3.	4.
$256.35	$275	$725	$743.65
145.24	146	854	854.76
321.75	281	719	678.25
286.44	675	325	713.56
156.82	284	716	843.18
244.31	552	448	755.69
986.24	496	504	13.76
275.46	285	715	724.54
383.27	628	372	616.73
152.10	397	603	847.90

(a) Find the sum of the four sums.

* TO THE PUPIL.—Remember that nothing short of absolute accuracy is of any value in such work as this.

Addition—Denominate Numbers.

45. Find the sum of 7 bu. 2 pk. 5 qt., 3 bu. 3 pk. 3 qt., 6 bu. 1 pk. 7 qt., and 9 bu. 3 pk. 5 qt.

Operation.
7 bu. 2 pk. 5 qt.
3 bu. 3 pk. 3 qt.
6 bu. 1 pk. 7 qt.
9 bu. 3 pk. 5 qt.
27 bu. 3 pk. 4 qt.

Explanation.

The sum of the number of quarts is 20; this is equal to 2 pecks and 4 quarts. Write the 4 quarts and add the 2 pecks to the pecks given in the second column.

The sum of the number of pecks is 11; this is equal to 2 bushels and 3 pecks. Write the 3 pecks, and add the 2 bushels to the bushels given in the third column.

The sum of the number of bushels is 27, which is written in its place.

The sum is 27 bu. 3 pk. 4 qt.

46. Problems.

1. Add.	2. Add.
6 bu. 2 pk. 6 qt.	5 bu. 0 pk. 7 qt.
4 bu. 2 pk. 2 qt.	4 bu. 1 pk. 6 qt.
5 bu. 2 pk. 2 qt.	3 bu. 0 pk. 1 qt.
6 bu. 3 pk. 7 qt.	5 bu. 0 pk. 5 qt.
4 bu. 3 pk. 3 qt.	1 bu. 1 pk. 2 qt.
8 bu. 2 pk. 6 qt.	2 bu. 3 pk. 3 qt.
7 bu. 0 pk. 5 qt.	4 bu. 2 pk. 4 qt.
5 bu. 1 pk. 4 qt.	2 bu. 0 pk. 6 qt.
8 bu. 0 pk. 6 qt.	6 bu. 0 pk. 5 qt.
7 bu. 3 pk. 2 qt.	1 bu. 3 pk. 2 qt.
3 bu. 3 pk. 3 qt.	2 bu. 2 pk. 0 qt.
6 bu. 1 pk. 7 qt.	7 bu. 0 pk. 2 qt.
7 bu. 2 pk. 0 qt.	3 bu. 2 pk. 2 qt.
2 bu. 3 pk. 6 qt.	3 bu. 1 pk. 2 qt.
6 bu. 1 pk. 6 qt.	3 bu. 2 pk. 1 qt.

(a) Find the sum of the two sums.

Algebraic Addition.

47. A coefficient is a number that indicates how many times a literal quantity* is to be taken; thus, in the expression $4ab$, 4 is the coefficient of ab.†

When no coefficient is expressed, it is understood that 1 is the coefficient; thus, in the expression $4a + b$, the coefficient of b is 1.

48. The terms of an algebraic expression are the parts that are separated by the sign + or −. There are three terms in the following: $ab + 3c + 4abc$. There are only two terms in the following: $8a \times 4b + 5a \div 6b$.

49. *Positive terms* are preceded by the plus sign.

50. *Negative terms* are preceded by the minus sign.

If no sign is expressed, the term is understood to be positive.

51. When the literal part of two or more terms is the same, the terms are said to be *similar*.

52. Problems.

Unite the terms in each of the following algebraic expressions into one equivalent term:

1. $5x + 3x + 2x =$
2. $4x + 5x - 3x =$
3. $6a - 2a + 4a =$
4. $3b + 4b - 2b =$
5. $4c + 3c - c =$
6. $4ab + 2ab + 3ab =$
7. $2ab + 5ab - 4ab =$
8. $3bc - 5bc + 6bc =$
9. $bx + 2bx + 3bx =$
10. $4b - 3b + 4b =$

53. Language Exercise.

Referring to the problems given above, use the following words in complete sentences:

Coefficient.	Terms.	Positive.
Negative.	Similar.	Literal.

* The word *quantity* in algebra means *number*. The expression literal quantity means number expressed by letters.

† The term coefficient is sometimes applied to the literal part of an expression; thus, in the expression abc, ab is the coefficient of c. Usually, however, the term coefficient has reference to the *numerical coefficient*.

Algebraic Addition.

54. Regarding the following *positive* numbers as representing *gains* and the *negative* numbers as representing *losses*, find the total gain (or loss) in each case; that is, find the algebraic sum of the numbers in each group:

1.	2.	3.	4.	5.	6.
70	85	45	$8a$	$9b$	$2ab$
25	-35	-65	$3a$	$-3b$	$-6ab$
95	50	-20			

NOTE.—The positive sums of Nos. 1, 2, 4, and 5 indicate actual gain; the negative sums of Nos. 3 and 6 indicate actual loss.

55. Regarding each of the following *positive* numbers as representing a *rise* and each of the *negative* numbers as representing a *fall* of the mercury in a thermometer, find the total rise (or fall) in each case; that is, find the algebraic sum of the numbers in each group:

1.	2.	3.	4.	5.	6.
16	12	8	$8a$	$7b$	$5c$
10	-4	4	$2a$	$-3b$	$3c$
5	6	-16	$3a$	$4b$	$-12c$
31	14	-4			

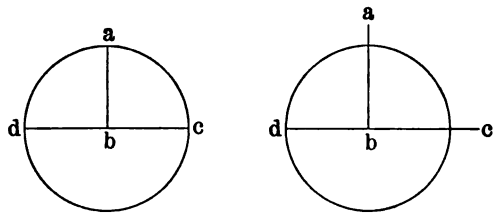
NOTE.—The positive sums of Nos. 1, 2, 4, and 5 indicate actual rise; the negative sums of Nos. 3 and 6 indicate actual fall.

56. PROBLEMS.

Find the sum:

1.	2.	3.
$12+4-6$	$3a+2b+c$	$2ab+6bc$
$3+5+2$	$2a-b+3c$	$5ab-2bc$
$4+6-3$	$a+5b-6c$	$3ab+bc$
$5-2+8$	$7a-2b+3c$	$-ab+3bc$

Geometry.

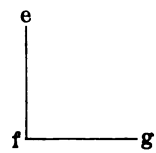


57. A **circle** is a plane figure bounded by a curved line, every point in which is equally distant from a point within, called the *center*.

58. The line that bounds a circle is a **circumference.**

59. A straight line passing through the center of a circle and ending in the circumference is a **diameter.**

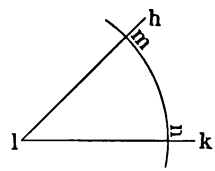
60. A straight line from the center of a circle to the circumference is a **radius.**



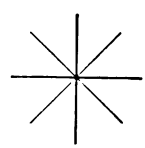
61. Any part of a circumference is an **arc.**

62. For the purpose of measurement, every circumference is considered as divided into 360 equal parts, called degrees.

63. Two lines meeting at a point form an **angle.** The point in which the two lines meet is the **vertex** of the angle.



64. Every angle may be regarded as having its vertex at the center of a circle, and the angle is measured by the part of the arc intercepted; thus, the angle *hlk* is measured by the arc *mn*.



65. The angle *efg* is an angle of 90 degrees, called also a right angle. The angle *abc* is an angle of 90 degrees. The angle *hlk* is an angle of 45 degrees.

66. All the angles about a point together equal four right angles.

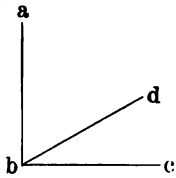
67. Miscellaneous Review.

1. The angle *abd* is an angle of about — degrees.

The angle *dbc* is an angle of — degrees.

The angle *abd* + the angle *dbc* = the angle *abc*.

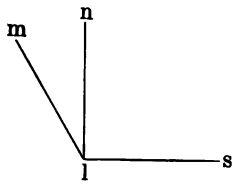
The angle *abc* is an angle of — degrees.



2. The angle *mln* is an angle of about — degrees.

The angle *nls* is an angle of about — degrees.

The angle *mln* + the angle *nls* = the angle *mls*.



The angle *mls* is an angle of about — degrees.

3. Earnings of ten persons for six days.

Copy the following figures and add by column and by line. Prove by comparing the sum of the sums of the columns with the sum of the sums of the lines. That pupil who can solve this problem without an error, on first trial, has taken an important step toward making himself valuable as an accountant.

	Mon.	Tues.	Wed.	Thurs.	Frid.	Sat.	Total.
A . . .	2.70	2.95	2.80	3.00	2.65	2.45	
B . . .	3.43	3.12	3.26	3.62	3.28	3.39	
C . . .	3.00	2.90	3.15	3.20	2.95	3.05	
D . . .	2.00	1.76	2.22	1.93	1.98	1.87	
E . . .	4.15	4.25	4.15	4.35	4.45	4.25	
F . . .	3.00	3.30	3.12	3.18	3.24	3.15	
G . . .	5.10	4.90	4.95	5.05	5.15	4.95	
H . . .	2.10	2.12	2.20	2.04	2.06	2.25	
I . . .	3.50	3.60	3.40	3.30	3.50	3.60	
K . . .	3.05	2.90	3.15	2.95	3.15	2.00	
Total . .							

SUBTRACTION.

68. Subtraction (in arithmetic) is the process of taking one number from (out of) another.

NOTE 1.—The word *number*, as here used, stands for *measured magnitude*, or *number of things*.

NOTE 2.—Subtraction (in general) is the process of finding the difference of two magnitudes.

69. The **minuend** is the number from which another number is taken.

70. The **subtrahend** is the number taken from another number.

71. The **difference** is the number obtained by subtracting.

72. The sign −, which is read *minus*, indicates that the number that follows the sign is to be taken from (out of) the number that precedes it; thus, 8 − 3, means, that 3 is to be taken from (out of) 8.

73. PRINCIPLES.

1. *Only like numbers can be subtracted.*

2. *The denomination of the difference is the same as that of the minuend and the subtrahend.*

74. PRIMARY FACTS OF SUBTRACTION.

There are eighty-one primary facts of subtraction which should be learned while learning the facts of addition. See Werner Arithmetic, Book II., p. 274, note.

75. Examples of Subtraction.

1.	2.	3.
2687	57.38	\$675.46
1298	28.146	\$282.75
1389	29.234	\$392.71

4.	5.	6.
576 pounds	25 bu. 3 pk. 5 qt.	$25a + 6b$
288 pounds	12 bu. 1 pk. 7 qt.	$10a + 2b$
288 pounds	13 bu. 1 pk. 6 qt.	$15a + 4b$

7. Tell which is the minuend, which the subtrahend, and which the difference, in each of the above examples.

76. *Observe* that in written problems in subtraction the subtrahend is usually written under the minuend and the difference under the subtrahend; and that, as in addition, the units of the same order are written in the same column.

1. In example 2, what figures represent units of the third decimal order? of the second integral order? of the first decimal order? of the first integral order?

2. In example 5, what figures represent units of the first integral order? of the second integral order?

77. *Observe* that in subtraction of denominate numbers the figures that stand for units of the same denomination and order are usually written in the same column.

1. In example 4, what figures represent tens of pounds? hundreds of pounds?

2. In example 5, what figures represent bushels and units of the first order?

78. *Observe* that in both addition and subtraction the decimal points, if there are any, usually appear in a column.

Subtraction—Simple Numbers.

79. Find the difference of 8274 and 5638.

Operation.

$$\begin{array}{r} 8274 \\ 5638 \\ \hline 2636 \end{array}$$

Explanation.

Eight is greater than 4. In the minuend, take one unit of the second order from the 7 units of the second order. This unit of the second order, combined with the 4 units of the first, makes 14 units of the first order. Eight units of the first order from 14 units of the first order leave 6 units of the first order. Three units of the second order from 6 (7 — 1) units of the second order leave 3 units of the second order. Six is greater than 2. In the minuend take one unit of the fourth order from the 8 units of the fourth order. This unit of the fourth order, combined with the 2 units of the third order, makes 12 units of the third order. Six units of the third order from 12 units of the third order leave 6 units of the third order. Five units of the fourth order from 7 (8 — 1) units of the fourth order leave 2 units of the fourth order.

The difference of 8274 and 5638 is 2636.

80. Problems.

1. From 35642 subtract 12456.
2. From 87544 subtract 64358.
3. From 90070 subtract 13256.
4. From the sum of 8539, 2647, 3984, 1461, 7353, 6016, and 2364, subtract 22364.
5. From the sum of 1352, 3425, 2640, 3724, 6575, 7360, and 6276, subtract 21352.
6. From 8 thousand 1 hundred 64, subtract 3 thousand 2 hundred 75.
7. From 6 thousand 7 hundred 25, subtract 1 thousand 8 hundred 36.
8. From seven thousand four hundred sixty-five, subtract two thousand three hundred fifty-four.

(a) Find the sum of the eight differences.

To the Pupil.—Do not allow yourself to make *one* error. Find the eight differences and their sum, accurately, *on first trial.*

Subtraction—Decimals.

81. Find the difference of 28.36 and 15.432.

Operation.	*Explanation.*
28.36	One unit of the second decimal order (1 from 6) equals
15.432	10 units of the third decimal order. Two from 10
12.928	leaves 8.
	Three from 5 (6 — 1) leaves 2.

Four is greater than 3. One unit of the first integral order (1 from 8) equals 10 units of the first decimal order. $10 + 3 = 13$. Four from 13 leaves 9.

Five from 7 (8 — 1) leaves 2.

One from 2 leaves 1.

The difference of 28.36 and 15.432 is 12.928.

82. PROBLEMS.

1. From 100 take .3456
2. From 100 take 5.246
3. From 100 take 44.236
4. From 100 take .6544
5. From 100 take 4.754
6. 100 – 44.764
7. 250 – 159.63
8. 250 – 36.75
9. 250 – 140.37
10. 250 – 163.25

(a) Find the sum of the ten differences.

83. MISCELLANEOUS.

1. The sum of two numbers is 3.7464; one of the numbers is 1.3521. What is the other number?
2. The difference of two numbers is 2.3254; the less number is 7.6746. What is the greater number?
3. The difference of two numbers is 2.3943; the greater number is 10. What is the less number?
4. The difference is 3.2678; the subtrahend is 2.1356. What is the minuend?
5. From 10 subtract 7.6744.

(b) Find the sum of the five results.

TO THE PUPIL.—Work with care. Make no errors. The sum of the results should be correct on first trial.

Subtraction—United States Money.

84. Find the difference of \$27.25 and \$14.51.

Operation.	*Explanation.*
\$27.25	One cent from 5 cents = 4 cents.
\$14.51	Five dimes from 12 dimes = 7 dimes.
\$12.74	Four dollars from 6 dollars (7 − 1) = 2 dollars.
	One ten-dollars from 2 ten-dollars = 1 ten-dollars.
	The difference of \$27.25 and \$14.51 is \$12.74.

85. Problems—Addition and Subtraction.*

Note.—The following represent bank accounts of six depositors for one day. Find the sum that each depositor has to his credit at the close of the day.

A.			B.		
Deposit		\$254.20	Deposit		\$175.30
Deposit		38.60	Check	\$30.50	
Check	\$12.50		Check	21.75	
Check	10.80		Deposit		54.20
Check	3.60		Check	18.34	
Check	5.40		Check	6.24	
Balance	——		Balance	——	

C.			D.		
Deposit		\$745.80	Deposit		\$824.70
Check	\$87.50		Check	\$69.50	
Check	89.20		Check	78.25	
Check	96.40		Check	81.66	
Check	94.60		Deposit		61.40
Deposit		45.80	Check	93.76	
Balance	——		Balance	——	

E. Deposit, \$1000. Checks, \$275, \$324, \$400.

F. Deposit, \$864. Checks, \$375, \$146, \$279.

(a) Find the amount of the six balances.

* That bank clerk who makes one error a day in carrying out his balances, which he does not himself discover and correct, would not retain his position.

Subtraction – Denominate Numbers.

86. Find the difference of 15 yd. 2 ft. 4 in. and 8 yd. 1 ft. 10 in.

Operation.	*Explanation.*
15 yd. 2 ft. 4 in. 8 yd. 1 ft. 10 in. 7 yd. 0 ft. 6 in.	Ten inches are more than 4 in.; 1 ft. (from the 2 ft.) equals 12 in.; 12 in. and 4 in. are 16 in.; 10 in. from 16 in. leave 6 in. One ft. from 1 ft. (2 — 1) leaves 0 ft.

Eight yd. from 15 yd. leave 7 yd.

The difference of 15 yd. 2 ft. 4 in. and 8 yd. 1 ft. 10 in. is 7 yd. 6 in.

87. Problems.

1. From 12 yd. 1 ft. 8 in. subtract 5 yd. 2 ft. 3 in.
2. From 8 yd. 2 ft. 6 in. subtract 5 yd. 1 ft. 10 in.
3. From 9 yd. 1 ft. subtract 2 yd. 2 ft. 7 in.
4. From 6 yd. 2 ft. 5 in. subtract 4 yd. 8 in.
5. From 7 yd. subtract 5 yd. 1 ft. 1 in.
6. From 7 yd. 1 ft. 4 in. subtract 4 yd. 9 in.
7. From 11 yd. 6 in. subtract 4 yd. 1 ft. 2 in.
8. From 10 yd. 2 ft. subtract 7 yd. 5 in.
9. From 13 yd. 7 in. subtract 5 yd. 2 ft. 4 in.
10. From 13 yd. subtract 4 yd. 1 ft. 11 in.

(a) Find the sum of the ten differences.

88. Problems.

1. How many days from April 25 to May 1?
2. How many days from April 25 to May 10?
3. How many days from April 25 to May 20?
4. How many days from April 25 to June 1?
5. How many days from April 25 to June 10?
6. How many days from April 25 to June 30?
7. How many days from April 25 to July 5?
8. How many days from April 25 to Sept. 10?

(b) Find the sum of the eight answers.

Algebraic Subtraction.

89. Regarding the following minuends as representing A's gain (or loss), and the subtrahends as representing B's gain (or loss), subtract B's from A's.*

	Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.
A,	80	60	20	$7a$	$-4b$	$-12c$
B,	30	-20	50	$2a$	$-6b$	$-6c$
	50	80	-30	$5a$	$2b$	$-6c$

NOTE.—The positive differences for Monday, Tuesday, Thursday, and Friday indicate that A's gain was greater (or his loss less) than B's. The negative differences for Wednesday and Saturday indicate that A's gain was less (or his loss greater) than B's.

90. Regard the following minuends as representing distances one boat sails from a given point, and the subtrahends as representing distances another boat sails from the same point. Distances sailed north are here represented by positive numbers, and distances sailed south by negative numbers. Find how far the first boat is from the second.

	1.	2.	3.	4.	5.	6.
1st B.,	75	85	15	$8a$	$-4b$	$-15c$
2nd B.,	25	-35	35	$2a$	$-9b$	$-5c$
	50	120	-20			

NOTE.—The positive differences in Nos. 1, 2, 4, and 5 indicate that the first boat is north of the second boat. The negative differences in Nos 3 and 6 indicate that the first boat is south of the second boat.

91. From the foregoing learn that the subtraction of a positive number gives the same result as the addition of an equal negative number; and the subtraction of a negative number the same result as the addition of an equal positive number. Hence the rule for algebraic subtraction: *Conceive the sign (or signs) of the subtrahend to be changed* (− to + and + to −), *then proceed as in addition.*

* Remember that positive numbers are to represent gains and negative numbers losses. See page 28, Art. 54.

Algebraic Subtraction.

1. A gained \$1200 and lost \$250; B gained \$500 and lost \$350. How much more was A's wealth increased by the two transactions than B's?

\$1200 − \$250	$12a - 5b$	$14a - 3b$
\$500 − \$350	$5a - 7b$	$6a - 5b$
\$700 + \$100		

2. C gained \$1500 and \$650; D gained \$600 and lost \$250. How much more was C's wealth increased by the two transactions than D's?

\$1500 + \$650	$15a + 13b$	$13a + 5b$
\$600 − \$250	$6a - 5b$	$4a - 3b$
\$900 + \$900		

3. E gained \$1300 and lost \$450; F gained \$400 and \$250. How much more was E's wealth increased by the two transactions than F's?

\$1300 − \$450	$13a - 9b$	$17a - 8b$
\$400 + \$250	$4a + 5b$	$4a + 6b$
\$900 − \$700		

4. G gained \$1200 and lost \$500; H gained \$900 and lost \$100. How much more was G's wealth increased by the two transactions than H's?*

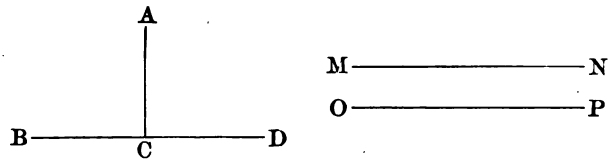
\$1200 − \$500	$12a - 5b$	$16a - 8b$
\$900 − \$100	$9a - b$	$4a - 2b$
\$300 − \$400		

5. Review the foregoing and observe that in every instance subtracting a positive number is equivalent to adding an equal negative number, and subtracting a negative number is equivalent to adding an equal positive number.

* The answer to this problem is, \$300 − \$400, or − \$100. Therefore, G gained − \$100 more than H, which means that his gains were actually \$100 less than H's.

Geometry.

92. Direction of Lines.



1. When one straight line meets another straight line in such a manner that two right angles are formed by the lines, the two lines are said to be *perpendicular* to each other.

2. Two lines side by side extending in the same direction are said to be *parallel.*

3. Of the lines given above:

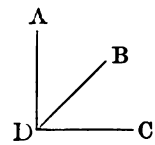
AC is ———— to *BD*
BD is ———— to *AC.*
MN is ———— to *OP.*
OP is ———— to *MN.*

4. A line extending in the direction of the *horizon* is said to be *horizon*tal. A line on the floor of the room is horizontal; a line on the ceiling is horizontal; a line on the blackboard, every point in which is equally distant from the floor, is horizontal. For convenience, lines drawn upon paper, that are parallel with the top and bottom of the paper, may be regarded as representing horizontal lines.

5. A piece of lead (plumbum), or other heavy material suspended by a cord, is called a plumb-line. A line in the direction of a plumb-line is said to be vertical. A vertical line is perpendicular to a horizontal line. Lines on the blackboard may or may not be vertical or horizontal. For convenience, lines drawn upon paper, that are parallel with the sides of the paper may be regarded as vertical lines.

93. Miscellaneous Reviews.

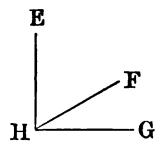
1. The angle formed by a vertical line meeting a horizontal line, is an angle of —— degrees.



2. An angle that is equal to one half of a right angle, is an angle of —— degrees.

3. The angle *ADB* is an angle of —— degrees.

4. The angle *BDC* is an angle of —— degrees.



5. If from a right angle, an angle of 30 degrees be taken, the remaining angle is an angle of —— degrees.

6. The angle *FHG* is an angle of —— degrees.

7. During the month of November, 1897, there were consumed at the Illinois Institution for the Education of the Blind, 64 loads of coal. The weight of each load in pounds is given below. Find the total weight.

6100	8020	5490	5190
8380	6860	6800	7130
4850	6230	6560	7090
8010	6780	6690	7790
7080	6980	5780	6810
6620	6240	6980	8600
6450	6420	5990	9100
6570	6310	4740	6740
7950	6300	5520	5380
4750	6530	3630	7640
8840	6950	4930	5650
7290	6980	5150	5900
4960	4920	6420	6200
8330	5880	6770	6620
6300	7030	6220	7170
7080	5160	6020	9210

MULTIPLICATION.

94. Multiplication is the process of taking a number (of things) a number of times.

NOTE 1.—The word *number* as first used in the above statement, stands for measured magnitude. The second word *number* does not stand for measured magnitude, but rather for pure number, representing simply the *times* the number (of things) is to be repeated.

95. The **multiplicand** is the number (of things) taken or repeated. *

96. The **multiplier** is the number that shows how many times the multiplicand is to be repeated.

97. The **product** is the number (of things) obtained by multiplying.

98. The sign, ×, which is read *multiplied by*, indicates that the number preceding the sign is a multiplicand, and the number following it, a multiplier.

NOTE.—For other uses of this sign, see Werner Arithmetic, Book II., page 274.

99. PRINCIPLES.

1. *The multiplier is always an abstract number.*
2. *The denomination of the product is always the same as that of the multiplicand.*

100. PRIMARY FACTS OF MULTIPLICATION.

There are sixty-four primary facts of multiplication. See Werner Arithmetic, Book II., p. 275.

* "The multiplicand, however written, must always be understood to express measured quantity; it is always concrete."—*Psychology of Number, McClellan & Dewey*, page 76.

101. Examples of Multiplication.

1.	2.	3.
4865	37.258	$375.42
3	3	3
14595	111.774	$1126.26

4.	5.	6.
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$
5	5	5
1820 tons.	309 yd. 0 ft. 1 in.	$40a - 30b$

7. Tell which is the multiplicand, which the multiplier, and which the product, in each of the above examples.

102. *Observe* that in each of the above examples the multiplier is a pure number.

103. *Observe* that in each of the above examples the denomination of the product is the same as the denomination of the multiplicand.

104. Multiplication and Addition Compared.

Find the sum of each of the following groups of numbers and compare the result with the product in the corresponding problem in article 101.

1.	2.	3.
4865	37.258	$375.42
4865	37.258	$375.42
4865	37.258	$375.42

4.	5.	6.
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$
364 tons	61 yd. 2 ft. 5 in.	$8a - 6b$

MULTIPLICATION—SIMPLE NUMBERS.

105. Find the product of 563 and 7.

Operation.

563
7
3941

Explanation.

Seven times 3 units of the first order are 21 units of the first order; they are equal to 1 unit of the first order and 2 units of the second order. Write the 1 unit of the first order and add the 2 units of the second order to the next partial product.

Seven times 6 units of the second order are 42 units of the second order; $42 + 2 = 44$; 44 units of the second order equal 4 units of the second order and 4 units of the third order. Write the 4 units of the second order and add the 4 units of the third order to the next partial product.

Seven times 5 units of the third order are 35 units of the third order; $35 + 4 = 39$; 39 units of the third order equal 9 units of the third order and three units of the fourth order. Write the 9 units of the third order and the 3 units of the fourth order.

The product of 563 and 7 is 3941.

106. Find the product of 3426 and 57.

Operation.

3426
57
23982
17130
195282

Explanation.

Seven times 3426 equals 23982. Fifty times 3246 equals 171300.* Fifty times the number plus 7 times the number equals 57 times the number. Therefore adding 23982 and 171300 gives 57 times 3426.

The product of 3426 and 57 is 195282.

107. PROBLEMS.

1. Multiply 3241 by 27.
2. Multiply 6759 by 27.
3. Multiply 4328 by 36.
4. Multiply 5672 by 36.
5. Multiply 3479 by 54.
6. $6521 \times 54 =$
7. $3572 \times 74 =$
8. $6428 \times 74 =$
9. $3521 \times 29 =$
10. $6479 \times 29 =$

(*a*) Find the sum of the ten products.

*Ten times five times a number equals fifty times the number. See Werner Arithmetic, Book II., p. 61.

Multiplication—Decimals.

108. Find the product of 728.37 and .6

Operation.	*Explanation.*
72⁷8.37 .6 437.022	To multiply by .6 means to take 6 times 1 tenth of the number. One tenth of 728.37 is 72.837. Six times 72.837 equals 437.022.

NOTE 1.—The separatrix is used to indicate the place of the decimal point in the number denoting one tenth of the multiplicand.

NOTE 2.—The decimal point should be written in the product when, in the process of multiplication, the place is reached where it belongs. Do not multiply all the figures and then attempt to determine the place of the point.

109. Find the product of 746.2 and .25

Operation.	*Explanation.*
7⁷46.2 .25 37.310 149.24 186.550	To multiply by .25 means to take 25 times 1 hundredth of the number. One hundredth of 746.2 is 7.462. Five times 7.462 equals 37.310. Twenty times 7.462 equals 149.24. 37.310 + 149.24 = 186.550. NOTE.—For direction as to the use of the separatrix, see Werner Arithmetic, Book II, p. 133, note.

110. *Observe* that when a multiplication of decimals is complete, the number of decimal places in the product is equal to the number of decimal places in the multiplicand and multiplier.

111. PROBLEMS.

1. Multiply 324.6 by .7
2. Multiply 324.6 by .27
3. Multiply 324.6 by 2.7
4. Multiply 324.6 by 27.
5. Multiply 324.6 by 5.4
6. 324.6 × 54 =
7. 324.6 × .48 =
8. 324.6 × 6.3 =
9. 324.6 × 3.07 =
10. 324.6 × .08 =

(*a*) Find the sum of the ten products.

Multiplication—United States Money.

112. Find the product of $34.50 and 53.4.

Operation.	*Explanation.**
$34.50 53.4	To multiply by 53.4, means to take 53 times the multiplicand plus 4 tenths of the multiplicand.
$13.800	One tenth of $34.50 is $3.45.
$103.50	Four tenths of $34.50 is $13.80.
$1725.0	Three times $34.50 is $103.50.
$1842.300	Fifty times $34.50 is $1725.
	The sum of these three partial products is $1842.30.

Practical application of the foregoing.

If one acre of land costs $34.50, how much will 53.4 acres cost?

One tenth of an acre costs $3.45.

Four tenths of an acre cost $13.80.

Three acres cost $103.50.

Fifty acres cost $1725.

53.4 acres cost $1842.30.

113. Complete the following bill, and find the amount of it.

INST. FOR THE BLIND,

1897. *To* GEO. E. SYBRANT, *Dr.*

Dec.	4	27 bbl. Apples	@	$2.25	
"	10	56 bush. Potatoes	@	.52	
"	12	13 bush. Beans	@	1.75	
"	16	34 bush. Turnips	@	.35	
"	18	50 bbl. Flour	@	4.90	
"	21	74 lb. Butter	@	.19	
"	22	53 lb. Tea	@	.42	
"	24	37 bush. Onions	@	.55	
"	31	58 lb. Ham	@	.14	

TO THE PUPIL.—Remember that any inaccuracy in solving business problems makes the work valueless. Accuracy ranks next in importance to integrity in the selection of an accountant.

Multiplication—Denominate Numbers.

114. Find the product of 3 tons 850 lbs. and 8.

Operation.	*Explanation.*
3 tons 850 lbs.	Eight times 850 lb. equals 6800 lb.
8	6800 lb. equals 3 tons 800 lb.
27 tons 800 lb.	Write the 800 lb. and add the 3 tons to the next partial product.

Eight times 3 tons equals 24 tons; 24 tons plus 3 tons equals 27 tons. 3 tons 800 lb. multiplied by 8 equals 27 tons 800 lb.

115. Problems.

1. If one side of a square garden measures 6 rd. 8 ft., what is the perimeter of the garden?

2. The circumference of a certain bicycle track is 13 rd. 12 ft. How far does the rider travel who goes around it 12 times?

3. The length of a rectangular field is 15 rd. 10 ft. and the width 9 rd. 8 ft. What is the perimeter of the field?

4. There is a walk 5 feet wide around a rectangular grass plat 3 rd. 6 ft. by 2 rd. 10 ft. What is the outside perimeter of the walk?

5. How far does the person travel who walks once around the grass plat described in problem 4, if he keeps his track in the center of the walk?

(*a*) Find the sum of the five answers.

116. Problems.

1. If a train moves on the average at the rate of a mile in 1 min. 25 sec., in how long a time will it move 325 miles?

2. If each of 56 grain bags contains 2 bush. 1 pk. 4 qt., how much grain in all?

3. If the circumference of a wagon wheel is 15 feet 6 inches, how far will the wagon move while the wheel revolves 1000 times?

Algebraic Multiplication.

117. Examples.

No. 1.

$$\begin{array}{r} 8+4-3 \\ 4 \\ \hline 32+16-12 \end{array}$$

No. 2.

$$\begin{array}{r} a+3b-4c \\ 4 \\ \hline 4a+12b-16c \end{array}$$

No. 3.

$$\begin{array}{r} 3a-2b+c \\ d \\ \hline 3ad-2bd+cd \end{array}$$

No. 4.

$$\begin{array}{r} 2a+3b-5c \\ 2d \\ \hline 4ad+6bd-10cd \end{array}$$

1. *Observe* that in the above examples we multiply each term of the multiplicand by the multiplier. As a matter of convenience it is best to begin with the term of the multiplicand on the left side and proceed from left to right.

2. Prove example No. 1. by uniting the terms of the multiplicand and comparing 4 times the number thus obtained with the number obtained by uniting the terms of the product.

3. Verify Example No. 2. by letting $a = 5$, $b = 3$, and $c = 2$.

4. Verify Example No. 3. by giving the following values to the letters: $a = 7$, $b = 4$, $c = 3$, $d = 5$.

5. Verify Example No. 4. by giving any values you may choose to each letter.

118. Problems.

1. Multiply $3ab - 2bc + 5c$ by $2d$.
2. Multiply $2ax + 4bx - y$ by 5.
3. Multiply $3bc + ab - bc$ by $3d$.
4. Multiply $x - y + z$ by $3ab$.
5. Multiply $ax + bx - cx$ by $2y$.
6. Verify each of the above problems by giving the following values to the letters: $a = 3$, $b = 2$, $c = 4$, $d = 5$, $x = 7$, $y = 6$, $z = 8$.

Algebraic Multiplication.

119. Exponent.

1. $a \times a$, or aa which means a multiplied by a, is usually written a^2. This is read a square or a second power.

2. b^3 (to be read b cube or b third power) means b taken three times as a factor. It is $b \times b \times b$.

3. a^4 (to be read a fourth power, or simply a fourth) means that a is taken four times as a factor. It is $a \times a \times a \times a$.

4. The small figure at the right of a letter tells the number of times the letter is to be used as a factor. The figure so used is called an exponent. When the exponent is 1, it is not usually expressed; thus, a means a^1.

120. Problems.

On the supposition that a = 2, b = 3, and c = 4, find the numerical value of each of the following expressions:

1. $a^2 + 2ab + b^2$
2. $3ab^2 + 5bc^2$
3. $4a^2b^2 + 3b^2c^2$
4. $2a^2b + 2ab^2$
5. $3b^2c^2 + 5ab$
6. $5a^2b - 2bc$
7. $a^3b^3 - c^3$
8. $a^2b^2 + c^2$
9. $a^2b^2c^2 - ab^3$
10. $2a^2b^2c^2$

(a) Find the sum of the numerical values of the above.

121. Examples.

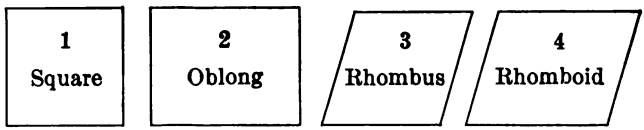
No. 1.	No. 2.
$4ax + 2by + c$	$3b^2x + 2by - c$
a^2	$2b^2$
$4a^3x + 2a^2by + a^2c$	$6b^4x + 4b^3y - 2b^2c$

1. *Observe* that in the above examples the product of the coefficients and the sum of the exponents is taken.

2. Verify each of the above examples by letting $a = 2$, $b = 3$, $c = 4$, $x = 5$, $y = 6$.

Geometry.

122. PARALLELOGRAMS.



1. Any side of any one of the above figures is parallel to the opposite side of the same figure. Hence the figures are called *parallelograms*.

2. Each of the above figures has four sides. Hence the figures are called *quadrilaterals*.

3. If all the sides of a figure are equal, the figure is said to be *equilateral*.

4. If all the angles of a parallelogram are right angles (angles of 90°) the figure is said to be *rectangular*.

5. Which of the above figures are equilateral?

6. Which of the above figures are rectangular?

7. Which of the above figures are not equilateral?

8. Which of the above figures are not rectangular?

9. Which of the above figures are parallelograms?

10. Which of the above figures are quadrilaterals?

11. Can you draw a quadrilateral that is not a parallelogram?

12. Is any one of the above figures an equilateral rectangular parallelogram?

13. *Observe* that every rectangular parallelogram has four right angles.

14. In a rhomboid or a rhombus two of the angles are less than right angles and two of them are greater than right angles. Convince yourself by cutting a rhomboid from paper and comparing it with rectangular figures that two of the angles of a rhomboid are as much less than two right angles as the other two are greater than two right angles.

123. Miscellaneous Reviews.

1. If one of the angles of a rhombus is an angle of 80 degrees, what is the number of degrees in each of the other angles?

2. Draw a rhomboid one of whose angles is an angle of 70; give the number of degrees in each of the other angles.

3. An oblong has four right angles. The angles of a rhomboid are together equal to how many right angles?

4. If an oblong is a feet long and b feet wide, the number of square feet in the area is ab.* If the side of a square is a, the number of square feet in its area is ————.

5. If a rectangular solid is a feet long, b feet wide, and c feet thick, the number of cubic feet in its solid contents is abc. If the side of a cube is a feet, the number of cubic feet in its solid contents is ————.

6. If a man earns b dollars each week and spends c dollars, in one week he will save ———— dollars; in 7 weeks he will save ———— dollars.

7. A framed picture, on the inside of the frame, is 18 in. by 22 in.; the frame is 4 inches wide. How many inches in the outside perimeter of the frame?

8. Think of two fields: one is 9 rd. by 16 rd.; the other is 12 rd. by 12 rd. How do the square rods of the two fields compare? How much more fence would be required to enclose one field than the other?

* This means *the product* of a and b. Observe that it is the number a (not a feet) that we multiply by the number b (not b feet). While it is probably true, (see foot-note, p. 41), that the multiplicand always expresses measured quantity, it is also true that we often find the product of two factors mechanically. Indeed this is what we usually do in all multiplication of abstract numbers. In this case we find the product of a and b and know from former observations that this number equals the number of square feet in the oblong.

DIVISION.

124. Division is (1) the process of finding how many times one number is contained in another number; or (2), it is finding one of the equal parts of a number.

NOTE 1.—The word *number* as used above, stands for measured magnitude.

125. The **dividend** is the number (of things) to be divided.

NOTE.—Since in multiplication the multiplicand and product must always be considered concrete (see footnote, p. 41), then in division, the dividend, and either the divisor or the quotient, must be so regarded.

126. The **divisor** is the number by which we divide.

NOTE.—The word *number* as used in Art. 126 may stand for measured magnitude or for pure number, according to the aspect of the division problem. In the problem $324 \div 6$, if we desire to find how many times 6 is contained in 324, the 6 stands for measured magnitude—a number of things. But if we desire to find one sixth of 324, then the 6 is pure number, and is the ratio of the dividend to the required quotient.

127. The **quotient** is the number obtained by dividing.

NOTE.—If the divisor is pure number the quotient represents measured magnitude. If the divisor represents measured magnitude the quotient is pure number.

128. The sign, $\div$, which is read *divided by*, indicates that the number before the sign is a dividend and the number following the sign, a divisor.

129. Examples in Division.

No. 1.	No. 2.
$5)$1565	5)$1565
313	$313

No. 3.	No. 4.
2 bush.)246 bush.	2)246 bush.
123	123 bush.

No. 5.	No. 6.
$2a)6ab + 8ac - 12a$	$2)6ab + 8ac - 12a$
$3b + 4c - 6$	$3ab + 4ac - 6a$

1. In Example No. 1, we are required to find ———— in 1565 dollars.*

2. In Example No. 2, we are required to find —— of 1565 dollars.†

3. In Example No. 3, we are required ————.

4. In Example No. 4, we are required ————.

5. In Example No. 5, we are required ————.

6. In Example No. 6, we are required ————.

Note.—Let it be observed that all the examples given on this page, indeed all division problems, *may* be regarded as requirements to find how many times one number of things is contained in another number of like things. Referring to Example No. 2 given above: If one were required to find one fifth of 1565 silver dollars, he might first take 5 dollars from the 1565 dollars, and put one of the dollars taken in each of five places. He might then take another five dollars from the number of dollars to be divided, and put one dollar with each of the dollars first taken. In this manner he would continue to distribute fives of dollars until all the dollars had been placed in the five piles. He would then count the dollars in each pile. Observe, then, that one fifth of 1565 dollars is as many dollars as $5 is contained times in $1565. It is contained 313 times; hence one fifth of 1565 dollars is 313 dollars.

It is not deemed advisable to attempt such an explanation as the foregoing with young pupils; but the more mature and thoughtful pupils may now learn that it is possible to solve all division problems by one thought process—finding how many times one number of things is contained in another number of like things. But if this method is adopted great care must be taken both in understanding the conditions of the problems and in the interpretation of the results obtained.

* Fill the blank with the words, *how many times five dollars are contained.*

† Fill the blank with the words, *one fifth.*

Division—Simple Numbers.

130. Find the quotient of 576 divided by 4.

"Short Division."

$$4)576$$
$$144$$

Explanation No. 1.

One fourth of 5 hundred is 1 hundred with a remainder of 1 hundred; 1 hundred equals 10 tens; 10 tens plus 7 tens are 17 tens. One fourth of 17 tens is 4 tens with a remainder of 1 ten; 1 ten equals 10 units; 10 units plus 6 units are 16 units. One fourth of 16 units is 4 units. Hence one fourth of 576 is 144.

Explanation No. 2.

Four is contained in 5 hundred, 1 hundred times, with a remainder of 1 hundred; 1 hundred equals 10 tens; 10 tens and 7 tens are 17 tens. Four is contained in 17 tens, 4 tens (40) times with a remainder of 1 ten; 1 ten equals 10 units; 10 units and 6 units are 16 units. Four is contained in 16 units 4 times.

Hence 4 is contained in 576, 144 times.

131. Find the quotient of 8675 divided by 25.

"Long Division."

$$\begin{array}{r} 25)8675(347 \\ 75 \\ \hline 117 \\ 100 \\ \hline 175 \\ 175 \\ \hline \end{array}$$

Explanation No. 1.

One twenty-fifth of 86 hundred is 3 hundred, with a remainder of 11 hundred; 11 hundred equal 110 tens. 110 tens plus 7 tens equal 117 tens. One twenty-fifth of 117 tens is 4 tens, with a remainder of 17 tens; 17 tens equal 170 units; 170 units plus 5 units equal 175 units. One twenty-fifth of 175 units is 7 units.

Hence one twenty-fifth of 8675 is 347.

To the Pupil.—Make another explanation of this process similar to Explanation No. 2, under Art. 130.

132. Problems.

1. $93492 \div 49$	6. $5904 \div 328$
2. $92169 \div 77$	7. $7693 \div 157$
3. $72855 \div 45$	8. $8190 \div 546$
4. $34694 \div 38$	9. $12960 \div 864$
5. $54875 \div 25$	10. $10950 \div 438$

(a) Find the sum of the ten quotients.

Division—Decimals.

133. Find the quotient of 785.6 divided by .5.

Operation

.5)785.6ᵛ5
1571.3

Explanation.

First place a separatrix (ᵛ) after that figure in the dividend that is of the same denomination as the right hand figure of the divisor—in this case, after the figure 6. Then divide, writing the decimal point in the quotient when, in the process of division, the separatrix is reached—in this case, after the figure 1.

It was required to find how many times 5 tenths are contained in 7856 tenths. 5 tenths are contained in 7856 tenths 1571 times. There are yet 15 hundredths to be divided. 5 tenths are contained in 15 tenths, 3 times; in 15 hundredths, 3 tenths of a time.

NOTE.—By holding the thought for a moment upon that part of the dividend which corresponds in denomination to the divisor, the place of the decimal point becomes apparent at once.

5 apples are contained in 7856 apples, 1571 times.
5 tenths are contained in 7856 tenths, 1571 times.

134. Solve and explain the following problems with special reference to the placing of the decimal point:

1. Divide 340 by .8	.8)340.0ᵛ
2. Divide 468.5 by .25	.25)468.50ᵛ
3. Divide 38.250 by 12.5	12.5)38.2ᵛ50
4. Divide 87 by 2.5	2.5)87.0ᵛ
5. Divide 546 by .75	.75)546.00ᵛ
6. Divide .576 by 2.4	2.4).5ᵛ76
7. 86 ÷ .375	8. 94.5 ÷ .8
9. 75 ÷ .15	10. 125 ÷ .5

(a) Find the sum of the ten quotients.

Division—United States Money.

135. Divide $754.65 by $.27.

Operation.

$.27)$754.65ᵛ(2795
54
214
189
256
243
135
135

Explanation.

This means, find how many times 27 cents are contained in 75465 cents. 27 cents are contained in 75465 cents, 2795 times.

PROBLEM.

At 27¢ a bushel, how many bushels of oats can be bought for $754.65? As many bushels can be bought, as $.27 is contained times in $754.65. It is contained 2795 times. Therefore, 2795 bushels can be bought.

136. Divide $754.65 by 27.

Operation.

27)$754.ᵛ65($27.ᵛ95
54
214
189
256
243
135
135

Explanation.

This means, find one 27th of $754.65. One 27th of $754.65 is $27.95.

NOTE.—One might find 1 27th of $754.65 by finding how many times $27 is contained in $754.65. See p. 52, Note.

PROBLEM.

If 27 acres of land are worth $754.65, how much is one acre worth?

137. Divide $754.65 by .27.

Operation.

.27($754 .65ᵛ($2795
54
214
189
256
243
135
135

Explanation.

This means find 100 27ths of $754.65. One 27th of $754.65 is $27.95. 100 27ths of $754.65 is $2795.

NOTE.—In practice we find one 27th of 100 times $754. 65.

PROBLEM.

If .27 of an acre of land is worth $754.65, how much is 1 acre worth at the same rate?

DENOMINATE NUMBERS.

138. Divide 46 rd. 12 ft. 8 in. by 4.

Operation.	*Explanation.*
4)46 rd. 12 ft. 8 in.	This means, find 1 fourth of 46 rd. 12 ft. 8 in.
11 rd. 11 ft. 5 in.	

One fourth of 46 rd. is 11 rd. with a remainder* of 2 rd.; 2 rd. equal 33 ft. 33 ft. plus 12 ft. equal 45 ft.

One fourth of 45 ft. equals 11 ft. with a remainder of 1 ft.; 1 ft. equals 12 in.; 12 in. plus 8 in. equals 20 in. One fourth of 20 in. equals 5 in.

One fourth of 46 rd. 12 ft. 8 in. equals 11 rd. 11 ft. 5 in.

PROBLEM.

The perimeter of a square garden is 46 rd. 12 ft. 8 in. How far across one side of it?

139. MISCELLANEOUS.

Tell the meaning of each of the following, solve, explain, and state in the form of a problem the conditions that would give rise to each number process.

1. Multiply 64 rd. 14 ft. 6 in. by 8.
2. Divide 37 rd. 15 ft. 4 in. by 5.
3. Divide \$675.36 by \$48.
4. Divide \$675.36 by 48.
5. Divide \$675.36 by .48
6. Divide \$675.36 by \$4.8
7. Divide \$675.36 by 4.8
8. Divide \$675.36 by \$.48
9. Multiply \$356.54 by .36
10. Multiply \$356.54 by 3.6
11. Multiply \$356.54 by 36.
12. Multiply \$275.56 by 2.25.
13. Multiply \$275.56 by $2\frac{1}{4}$.
14. Can you multiply by a number of dollars?
15. Can you divide by a number of dollars?

* The word remainder in this connection suggests incomplete division. When the division is complete there can be no remainder.

Algebraic Division.

140. Examples.

No. 1.

$$4)12+5\times4-8$$
$$3+5-2$$

No. 2.

$$a)12a+2ab+4a$$
$$12+2b+4$$

No. 3.

$$2)2^3+3\times2^2-2$$
$$2^2+3\times2-1$$

No. 4.

$$b)ab^3+cb^2+3b$$
$$ab^2+cb+3$$

1. *Observe* that in the above examples we divide each term of the dividend by the divisor.

2. Prove Nos. 1 and 3, by (1) reducing each dividend to its simplest form, (2) dividing it so reduced, by the divisor, and (3) comparing the result with the quotient reduced to its simplest form.

3. Verify No. 2, by letting $a = 3$, and $b = 5$.

4. Verify No. 4, by letting $a = 3$, $b = 5$, and $c = 7$.

141. $(6\times a\times a\times a\times a\times a)\div(2\times a\times a)=6a^5\div2a^2=3a^3$.

Observe that to divide one algebraic term by another we must find the *quotient of the co-efficients* and the *difference of the exponents.*

142. Problems.

1. $6a^5b\div2a=$
2. $4a^4b^2\div2a=$
3. $8a^3b^3\div2a=$
4. $10a^2b^4\div2a=$

5.

$$2a)6a^5b+4a^4b^3-8a^3b^3+10a^2b^4$$

6. Verify Problem 5, by letting $a = 3$, and $b = 5$.

Algebraic Division.

143. Problems.

1. Divide $4a^3x + 8a^2x^2 + 6ax^3$ by $2ax$.
2. Multiply the quotient of problem 1, by $2ax$.
3. Verify problems 1 and 2, by letting $a = 2$, and $x = 3$.

4. Divide $3ab^3 + 6a^2b^3 + 9a^3b$ by $3ab$.
5. Multiply the quotient of problem 4, by $3ab$.
6. Verify problems 4 and 5, by letting $a = 3$, and $b = 5$.

7. Divide $2x^3y + x^2y^2 - xy^3$ by xy.
8. Multiply the quotient of problem 7, by xy.
9. Verify problems 7 and 8, by letting $x = 2$, and $y = 3$.

10. Divide $5a^3y^2 - 2a^2y^3 + a^2y^4$ by a^2y.
11. Multiply the quotient of problem 10, by a^2y.
12. Verify problems 10 and 11, by letting $a = 1$, and $y = 2$.

13. Divide $3b^4x + b^3x^2 - 3b^3x^3$ by bx.
14. Multiply the quotient of problem 13, by bx.
15. Verify problems 13 and 14, by letting $b = 3$, and $x = 4$.

Observe that when the divisor is a positive number, each term of the quotient has the same sign as the term in the dividend from which it was derived.

$2)\underline{8 - 6}$ One half of $+ 8$ is $+ 4$; one half of $- 6$ is $- 3$.
$4 - 3$

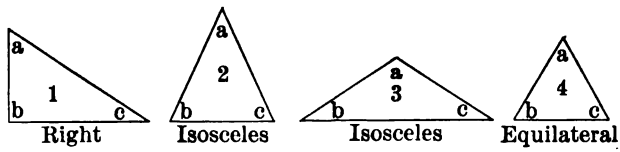
16.

$$2x)\underline{4x^5 - 6x^4 + 8x^3 - 2x^2 + 6x.}$$

17. Verify by letting $x = 2$.

Geometry.

144. TRIANGLES.



1. A triangle has —— sides and —— angles.

2. A right triangle has one right angle ; that is, one angle of —— degrees.

3. An isosceles triangle has two angles that are equal and two sides that are equal.

4. An equilateral triangle has equal sides and equal angles.

5. Find in the above figures all the angles that are right angles; all that are less than right angles; all that are greater than right angles.

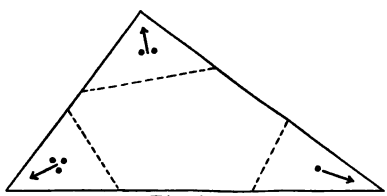


Fig. 5

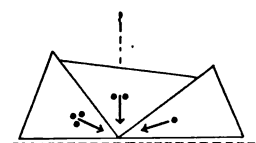
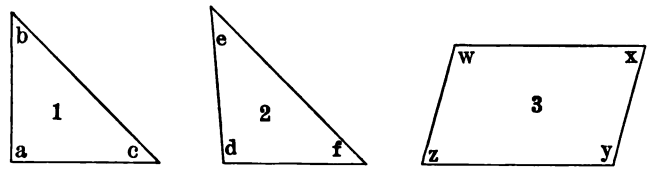


Fig. 6

6. Cut from paper a triangle similar to the one shown in Fig. 5. Then cut it into parts as shown by the dotted lines. Re-arrange the 3 angles of the triangle as shown in Fig. 6. Compare the sum of the 3 angles with 2 right angles as shown in Fig. 6. Convince yourself that the three angles of this triangle are together equal to two right angles.

7. Cut other triangles and make similar comparisons, until you are convinced that *the sum of the angles of any triangle is equal to two right angles.*

145. Miscellaneous Review.



1. If in figure 1, the angle a is a right angle, and the angle b is equal to the angle c, the angle b is an angle of how many degrees?

2. If in figure 2, the angle d is an angle of 95° and the angle e is an angle of 40°, the angle f is an angle of how many degrees?

3. If in figure 3, the angle x is an angle of 75°, the angle w is an angle of how many degrees?

4. If in an oblong there are ab square feet, and the oblong is a feet long, it is ——— feet wide. $ab \div a =$

5. If in a rectangular solid there are abc cubic feet, and the solid is a feet long and b feet wide, it is ——————— feet thick. $abc \div ab =$

6. Verify problems 4 and 5 by letting $a = 3$, $b = 4$, and $c = 2$.

7. There is a field that contains 1736 square rods; it is 28 rods long. How wide is the field?

8. There is a solid that contains 4320 cubic inches; it is 24 inches long and 15 inches wide. How thick is the solid?

9. How many square inches of surface in the solid described in problem 8?

10. How many inches in the perimeter of the largest face of the solid described in problem 8?

PROPERTIES OF NUMBERS.

To the Teacher: Under this head, *number in the abstract* is discussed with little or no distinction between *numbers of things* and *pure number*. It is dissociation and generalization without which there could be little progress in the "science of number" or in the "art of computation."

146. Every number is *fractional*, *integral*, or *mixed*.

1. A **fractional number** is a number of the equal parts of some quantity considered as a unit; as, $\frac{3}{5}$, .9, 5 sixths.

2. An **integral number** is a number that is not, either wholly or in part, a fractional number; as, 15, 46, ninety-five.

3. A **mixed number** is a number one part of which is integral and the other part fractional; as, $5\frac{1}{2}$, 27.6, $274\frac{5}{8}$.

147. An **exact divisor** * of a number is a number that is contained in the number an integral number of times.

5 is an exact divisor of 15.
5 is not an exact divisor of 1.5.
$16\frac{2}{3}$ is an exact divisor of 100.

148. Every integral number is *odd* or *even*.

1. An **odd number** is a number of which two is not an exact divisor; as, 7, 23, 141.

2. An **even number** is a number of which two is an exact divisor; as, 8, 24, 142.

*Note.—An exact divisor of a number is sometimes called an **aliquot part** of the number.

Properties of Numbers.

149. Every integral number is *prime* or *composite.*

1. A **prime number** is an integral number that has no exact integral divisors except itself and 1; as, 23, 29, 31, etc.

2. Is *two* a prime number? *three? nine?*

3. Name the prime numbers from 1 to 97 inclusive*. Find their sum.

4. A **composite number** is an integral number that has one or more integral divisors besides itself and 1; as, 6, 8, 9, 10, 12, 14, 15, etc.

5. Name the composite numbers from 4 to 100 inclusive. Find their sum.

6. Is *eight* a composite number? *eleven? fifteen?*

(*a*) Find the sum of the results of problems No. 3 and No. 5.

150. To find whether an integral number is *prime* or *composite.*

1. Is the number 371 prime or composite?

Operation.		*Explanation.*
2)371	5)371	Beginning with 2 (the smallest prime number except the number 1) it is found by trial not to be an exact divisor of 371.
185+	74+	
3)371	7)371	3 is not an exact divisor of 371.
123+	53.	5 is not an exact divisor of 371.
		7 is an exact divisor of 371. Therefore 371 is a composite number, being composed of 53 sevens, or of 7 fifty-threes. †

Observe that we use as trial divisors only prime numbers. If 2 is not an exact divisor of a number, neither 4 nor 6 can be. Do you see why?

* There are 26 prime numbers less than 100.

† Note the similarity of the words *composed* and *composite.*

Properties of Numbers.

2. Is the number 397 prime or composite?

Operation.

2)397	3)397
198+	132+
5)397	7)397
79+	56+
11)397	13)397
36+	30+
17)397	19)397
23+	20+

Explanation.

By trial it is found that neither 2, 3, 5, 7, 11, 13, 17 or 19 is an exact divisor of 397.

No composite number between 2 and 19 can be an exact divisor of 397: for since one 2 is not an exact divisor of the number, several 2's, as 4, 6, 8, 12, etc., cannot be; since one 3 is not an exact divisor of the number, several 3's, as 6, 9, 12, etc., cannot be; since one 5 is not an exact divisor of the number, several 5's, as 10 and 15 cannot be; since one 7 is not an exact divisor of the number, two 7's (14) cannot be.

No number greater than 19 can be an exact divisor of the number; for if a number greater than 19 were an exact divisor of the number the quotient (which also must be an exact divisor) would be less than 20. But it has already been proved that no integral number less than 20 is an exact divisor of 397. Therefore 397 is a prime number.

Observe that in testing a number to determine whether it is prime or composite, we take as trial divisors, prime numbers only, beginning with the number two.

Observe that as the divisors become greater, the quotients become less, and that *we need make no trial by which a quotient will be produced that is less than the divisor.*

3. Determine by a process similar to the foregoing, whether each of the following is prime or composite: 127, 249, 257, 371.

151. Any divisor* of a number may be regarded as a factor of the number. An exact integral divisor of a number is an integral factor of the number.

* NOTE.—The word *factor* is often loosely used for *integral factor.*

Properties of Numbers.

152. Prime Factors.

1. An integral factor that is a prime number is a **prime factor.**

5 is a prime factor of 30.
7 is a prime factor of ——— and ———.
3 is a prime factor of ——— and ———.
2 and 3 are prime factors of ——— and ——— and ———.
3 and 5 are prime factors of ——— and ——— and ———.

2. Resolve 105 into its prime factors.

Operation.

$$\begin{array}{r|l} 5 & 105 \\ \hline 3 & 21 \\ \hline & 7 \end{array}$$

Explanation. Since the prime number 5 is an exact divisor of 105, it is a prime factor of 105. Since the prime number 3 is an exact divisor of the quotient (21), it is a prime factor of 21 and 105.

Since 3 is contained in 21 exactly 7 times, and since 7 is a prime number, 7 is a prime factor of 21 and of 105. Therefore the prime factors of 105 are 5, 3, and 7.

Observe that if 7 and 3 are prime factors of 21 they must be prime factors of 105, for 105 is made up of 5 21's. 7 is contained 5 times as many times in 105 as it is in 21.

Observe that every composite number is equal to the product of its prime factors.

$105 = 5 \times 3 \times 7$. $18 = 3 \times 3 \times 2$.

3. Resolve each of the following numbers into its prime factors: 224, 15, 21, 6. Then prove that the continued product of the numbers is equal to the continued product of all their factors.

Observe that 2 times 3 times a number equals 6 times the number; 3 times 5 times a number equals 15 times the number, etc.

Observe that instead of multiplying a number by 21, it may be multiplied by 3 and the product thus obtained by 7, and the same result be obtained as would be obtained by multiplying the number by 21. Why?

Properties of Numbers.

153. Multiples, Common Multiples, and Least Common Multiples.

1. A **multiple** of a number is an integral number of times the number.

30 and 35 and 40 are multiples of 5.
16 and 20 and 32 are multiples of 4.

2. A **common multiple** of two or more numbers is an integral number of times each of the numbers.

30 is a common multiple of 5 and 3.
40 is a common multiple of 8 and 10.
—— is a common multiple of 9 and 6.
—— is a common multiple of 8 and 12.

3. A common multiple of two or more integral numbers contains all the prime factors found in every one of the numbers, and may contain other prime factors.

$48 = 2 \times 2 \times 2 \times 2 \times 3$. $150 = 2 \times 3 \times 5 \times 5$. A common multiple of 48 and 150 must contain four 2's, one 3, and two 5's. It may contain other factors.

$2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 = 1200.$
$2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 2 = 2400.$
$2 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 2 \times 3 = 7200$

1200, 2400 and 7200 are common multiples of 48 and 150.

4. The **least common multiple** (l. c. m.) of two or more numbers is the least number that is an integral number of times each of the numbers.

40 and 80 and 120 are common multiples of 8 and 10; but 40 is the least common multiple of 8 and 10.

5. The least common multiple of two or more numbers contains all the prime factors found in every one of the numbers, and no other prime factors.

Properties of Numbers.

$36 = 2 \times 2 \times 3 \times 3$. $120 = 2 \times 2 \times 2 \times 3 \times 5$. The l. c. m. of 36 and 120 must contain three 2's, two 3's, and one 5. $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$, l. c. m. of 36 and 120.

6. To find the l. c. m. of two or more numbers: *Resolve each number into its prime factors. Take as factors of the l. c. m. the greatest number of 2's, 3's, 5's, 7's, etc., found in any one of the numbers.*

EXAMPLE.

Find the l. c. m. of 24, 35, 36, and 50.

OPERATION.

$$24 = 2 \times 2 \times 2 \times 3.$$
$$35 = 5 \times 7.$$
$$36 = 2 \times 2 \times 3 \times 3.$$
$$50 = 2 \times 5 \times 5.$$
$$2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 12600, \text{ l. c. m.}$$

EXPLANATION.

24 has the greatest number of 2's as factors.
36 has the greatest number of 3's as factors.
50 has the greatest number of 5's as factors.
35 is the only number in which the factor 7 occurs.

There must be as many 2's among the factors of the l. c. m. as there are 2's among the factors of 24; as many 3's as there are 3's among the factors of 36; as many 5's as there are 5's among the factors of 50; as many 7's as there are 7's among the factors of 35; that is, three 2's, two 3's, two 5's, and one 7.

Find the l. c. m.

7. Of 48 and 60.
8. Of 60 and 75.
9. Of 50 and 60.
10. Of 30 and 40.
11. Of 20, 30, and 40.
12. Of 40, 50, and 60.
13. Of 24, 48, and 36.
14. Of 25, 35, and 40.

Algebra—Parentheses.

154. When an expression consisting of two or more terms is to be treated as a whole, it may be enclosed in a parenthesis.

$$\begin{cases} 12 + (5 + 3) = ? \\ 12 + 5 + 3 = ? \end{cases} \qquad \begin{cases} 7a + (3a + 2a) = ? \\ 7a + 3a + 2a = ? \end{cases}$$

Observe that removing the parenthesis makes no change in the results.

$$\begin{cases} 12 - (5 + 3) = ? \\ 12 - 5 - 3 = ? \end{cases} \qquad \begin{cases} 7a - (3a + 2a) = ? \\ 7a - 3a - 2a = ? \end{cases}$$

Observe the change in signs made necessary by the removal of the parenthesis.

$$\begin{cases} 12 - (5 - 3) = ? \\ 12 - 5 + 3 = ? \end{cases} \qquad \begin{cases} 7a - (3a - 2a) = ? \\ 7a - 3a + 2a = ? \end{cases}$$

Observe the change in signs made necessary by the removal of the parenthesis.

A careful study and comparison of the foregoing problems will make the reasons for the following apparent:

I. *If an expression within a parenthesis is preceded by the plus sign, the parenthesis may be removed without making any changes in the signs of the terms.*

II. *If an expression within a parenthesis is preceded by a minus sign, the parenthesis may be removed; but the sign of each term in the parenthesis must be changed; the sign + to −, and the sign − to +.*

155. Problems.

(1) Remove the parenthesis, (2) change the signs if necessary, (3) combine the terms.

1. $15 - (6 + 4) =$
2. $18 + (4 - 3) =$
3. $27 - (8 + 3) =$
4. $45 + (12 - 3) =$
5. $75 - (18 + 27)$
6. $15b - (12b - 4b) =$
7. $18c + (9c - 3c) =$
8. $24d - (5d + 3d) =$
9. $36x - (5x + 4x) =$
10. $45y + (8y + 7y) =$

Algebra—Parentheses.

156. Multiplying an expression enclosed in a parenthesis.

$6(7 + 4) = ?$* $\qquad 6(7a + 4b) = ?$

$6(a + b) = ?$ Ans. $6a + 6b$.

$a(b + c) = ?$ Ans. $ab + ac$.

Observe that in multiplying the sum of two numbers by a third number, the sum may be found and multiplied; or each number may be multiplied and the sum of the products found.

1. In the last three examples given above, let $a = 5$, $b = 3$, and $c = 2$; then perform again the operations indicated, and compare the results with those obtained when the letters were employed.

$6(7 - 4) = ?$† $\qquad 6(7a - 4b) = ?$

$6(a - b) = ?$ Ans. $6a - 6b$.

$a(b - c) = ?$ Ans. $ab - ac$.

Observe that in multiplying the difference of two numbers by a third number, the difference may be found and multiplied; or each number may be multiplied and the difference of the products found.

2. In the last three problems given above, let $a = 5$, $b = 3$, and $c = 2$; then perform again the operations indicated and compare the results with those obtained when the letters were employed.

157. PROBLEMS.

If $a = 5$, $b = 3$, and $c = 2$, find the value of the following:

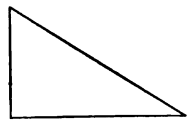
1. $3(a + b) - 2(b + c)$.
2. $4(a + 2b) - 3(b - c)$.
3. $2(2a - b) + 2(2b - c)$.

* This means, that the sum of 7 and 4 is to be multiplied by six; or that the sum of six 7's and six 4's is to be found.

† This means, that the difference of 7 and 4 is to be multiplied by six; or that the difference of six 7's and six 4's is to be found.

Geometry.

158. Triangles Continued.

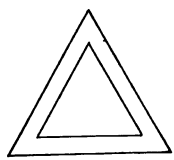
RIGHT  TRIANGLE.

1. The sum of the angles of any triangle is equal to —— right angles or —— degrees.*

2. In a right triangle there is one right angle. The other two angles are together equal to ————.

3. In a right triangle one of the angles is an angle of 40°. How many degrees in each of the other two angles? Draw such a triangle.

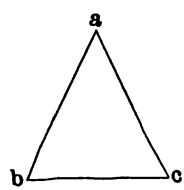
4. Convince yourself by drawings and measurements that every equilateral triangle is equiangular.

EQUILATERAL TRIANGLES.  EQUIANGULAR TRIANGLES.

5. Note that in every equiangular triangle each angle is one third of 2 right angles. So each angle is an angle of —— degrees.

6. If any one of the angles of a triangle is greater or less than 60, can the triangle be equiangular? Can it be equilateral?

7. If angle *a* of an isosceles triangle measures 50°, how many degrees in angle *b*? in angle *c*?



* See p. 59.

159. Miscellaneous Review.

1. I am thinking of a right triangle one of whose angles measures 32°. Give the measurements of the other two angles. Draw such a triangle.

2. I am thinking of an isosceles triangle; the sum of its two equal angles is 100°. Give the measurement of its third angle. Draw such a triangle.

3. Let a equal the number of degrees in one angle of a triangle and b equal the number of degrees in another angle of the same triangle; then the number of degrees in the third angle is $180° - (a + b)$. If a equals 30, and b equals 45, how many degrees in the third angle? Draw such a triangle.

4. Name four multiples of 16.

5. Name three common multiples of 16 and 12.

6. Name the *least* common multiple of 16 and 12.

7. Find the sum of all the prime numbers from 101 to 127 inclusive.

8. Find the prime factors of 836.

9. With the prime factors of 836 in mind or represented on the blackboard, tell the following:

(*a*) How many times is 19 contained in 836?

(*b*) How many times is 209, (11×19), contained in 836?

(*c*) How many times is 418, $(19 \times 11 \times 2)$, contained in 836?

160. Problems.

Find the l. c. m.

1. Of 18 and 20.
2. Of 13 and 11.
3. Of 24 and 32.
4. Of 16 and 38.
5. Of 46 and 86.
6. Of 36, 72, and 24.
7. Of 45, 81, and 27.
8. Of 33, 55, and 88.
9. Of 45, 65, and 85.
10. Of 3, 5, 7, and 11.

(*a*) Find the sum of the ten results.

DIVISIBILITY OF NUMBERS.

161. Numbers exactly divisible by 2; by $2\frac{1}{2}$; by $3\frac{1}{3}$; by 5; by 10.

1. An integral number is exactly divisible by 2 if the right-hand figure is 0, or if the number expressed by its right-hand figure is exactly divisible by 2.

EXPLANATORY NOTE. — Every integral number that may be expressed by two or more figures may be regarded as made up of a certain number of tens and a certain number (0 to 9) primary units, thus: 485 is made up of 48 tens and 5 units; 4260 is made up of 426 tens and 0 units; 27562 is made up of 2756 tens and 2 units. But ten is exactly divisible by 2; so any number of tens, or any number of tens plus any number of twos, is exactly divisible by 2.

2. Tell which of the following are exactly divisible by 2, and why: 387, 5846, 2750, 2834.

3. Any number, integral or mixed, is exactly divisible by $2\frac{1}{2}$, if the part of the number expressed by figures to the right of the tens' figure is exactly divisible by $2\frac{1}{2}$.

4. Show why the statement made in No. 3 is correct, employing the thought process given in the "Explanatory Note" above.

5. Tell which of the following are exactly divisible by $2\frac{1}{2}$, and why: 485, 470, 365, $472\frac{1}{2}$, $3847\frac{1}{2}$.

6. Any number, integral or mixed, is exactly divisible by $3\frac{1}{3}$, if the part of the number expressed by figures to the right of the tens' figure is exactly divisible by $3\frac{1}{3}$. Show why.

7. Tell which of the following are exactly divisible by $3\frac{1}{3}$, and why: 780, $283\frac{1}{3}$, $576\frac{2}{3}$, 742, 80.

Divisibility of Numbers.

8. Any integral number is exactly divisible by 5, if its right-hand figure is 0 or 5. Show why and give examples.

9. Any integral number is exactly divisible by 10 if its right-hand figure is ——.

162. Problems.

1. How many times is $2\frac{1}{2}$ contained in $582\frac{1}{2}$?*
2. How many times is $2\frac{1}{2}$ contained in 375?
3. How many times is $2\frac{1}{2}$ contained in $467\frac{1}{2}$?
4. How many times is $2\frac{1}{2}$ contained in 4680?
5. How many times is $3\frac{1}{3}$ contained in $786\frac{2}{3}$?†
6. How many times is $3\frac{1}{3}$ contained in $543\frac{1}{3}$?
7. How many times is $3\frac{1}{3}$ contained in 8640?
8. How many times is 5 contained in 3885?‡
9. How many times is 5 contained in 1260?
10. How many times is 2 contained in 8646?

163. Numbers exactly divisible by 25; by $33\frac{1}{3}$; by $12\frac{1}{2}$; by $16\frac{2}{3}$; by 20; by 50.

1. Any integral number is exactly divisible by 25 if its two right-hand figures are zeros or if the part of the number expressed by its two right-hand figures is exactly divisible by 25.

Explanatory Note.—Every integral number expressed by three or more figures may be regarded as made up of a certain number of hundreds and a certain number (0 to 99) primary units; thus 4624 is made up of 46 hundreds and 24 units; 38425 is made up of 384 hundreds and 25 units; 8400 is made up of 84 hundreds and 0 units. But a hundred is exactly divisible by 25; so any number of hundreds, or any number of hundreds plus any number of 25's is exactly divisible by 25.

*$2\frac{1}{2}$ is contained in $582\frac{1}{2}$, $(4 \times 58) + 1$ times. Why?
†$3\frac{1}{3}$ is contained in $786\frac{2}{3}$, $(3 \times 78) + 2$ times. Why?
‡5 is contained in 3885, $(2 \times 388) + 1$ times. Why?

Divisibility of Numbers.

2. Tell which of the following are exactly divisible by 25, and why: 37625, 34836, 27950, 38575.

3. Every number, integral or mixed, is exactly divisible by $33\frac{1}{3}$, if that part of the number expressed by the figures to the right of the hundreds' figure is exactly divisible by $33\frac{1}{3}$.

4. Show why the statement made in No. 3 is correct, employing the thought process given in the "Explanatory Note" under No. 1 on the preceding page.

5. Tell which of the following are exactly divisible by $33\frac{1}{3}$, and why: $36466\frac{2}{3}$, 2375, $46833\frac{1}{3}$, 38900, 46820.

6. Any number, integral or mixed, is exactly divisible by $12\frac{1}{2}$, if the part of the number expressed by the figures to the right of the hundreds' figure, is exactly divisible by $12\frac{1}{2}$. Show why.

7. Tell which of the following are exactly divisible by $12\frac{1}{2}$, and why; 375, $837\frac{1}{2}$, 6450, 4329, $7467\frac{1}{2}$, $3487\frac{1}{2}$, 3425.

8. Any number, integral or mixed, is exactly divisible by $16\frac{2}{3}$, if.....

9. Tell which of the following are exactly divisible by $16\frac{2}{3}$, and why: $4633\frac{1}{3}$, 5460, 2350, 37400, $27583\frac{1}{3}$, $25416\frac{2}{3}$.

10. Any integral number is exactly divisible by 20 if the number expressed by its two right-hand figures is exactly divisible by 20. Show why.

11. Tell which of the following are exactly divisible by 20, and why: 3740, 2650, 3860, 29480, 3470.

12. Tell which of the following are exactly divisible by 50, and why: 2460, 3450, 6800, 27380, 25450.

Divisibility of Numbers.

164. Problems.

1. How many times is 25 contained in 2450? *
2. How many times is 25 contained in 3775?
3. How many times is $33\frac{1}{3}$ contained in $4666\frac{2}{3}$? †
4. How many times is $33\frac{1}{3}$ contained in $3433\frac{1}{3}$?
5. How many times is $12\frac{1}{2}$ contained in $4737\frac{1}{2}$? ‡
6. How many times is $12\frac{1}{2}$ contained in $3662\frac{1}{2}$?
7. How many times is $16\frac{2}{3}$ contained in $2533\frac{1}{3}$?
8. How many times is $16\frac{2}{3}$ contained in 4550?

165. Numbers exactly divisible by 9.

1. Any number is exactly divisible by 9 if the sum of its digits is exactly divisible by 9.

Explanatory Note.—Any number more than nine is a certain number of nines and as many over as the number indicated by the sum of its digits. Thus, 20 is two nines and 2 over; 41 is four nines and 4 + 1 over; 42 is four nines and 4 + 2 over; 200 is twenty-two nines and 2 over; 300 is thirty-three nines and 3 over; 320 is a certain number of nines and 3 + 2 over; 321 is a certain number of nines and 3 + 2 + 1 over.

326 is a certain number of nines and 3 + 2 + 6 over; but 3 + 2 + 6 = 11, or another nine and 2 over.

2. Read the "Explanatory Note" carefully, and tell which of the following are exactly divisible by 9: 3256, 4266, 2314, 2574.

166. Problems.

1. 4625 is a certain number of 9's and ——— over.
2. 3526 is a certain number of 9's and ——— over.
3. 2154 is a certain number of 9's and ——— over.

* 25 is contained in 2450 (4 × 24) + 2 times. Why?
† $33\frac{1}{3}$ is contained in $4666\frac{2}{3}$ (3 × 46) + 2 times. Why?
‡ $12\frac{1}{2}$ is contained in $4737\frac{1}{2}$ (8 × 47) + 3 times Why?

Divisibility of Numbers.

167. Prime Factors and Exact Divisors.

1. Any integral number is exactly divisible by each of its prime factors and by the product of any two or more of its prime factors. Thus, 30, $(2 \times 3 \times 5)$, is exactly divisible by 2, by 3, by 5, and by (2×3), 6, and by (2×5), 10, and by (3×5), 15.

2. The exact integral divisors of 36, $(2 \times 2 \times 3 \times 3)$, are 2, 3, —, —, —, and ——.

168. Prime Factors, Common Divisors, and Greatest Common Divisors.

1. Any prime factor or any product of two or more prime factors common to two or more numbers is a common divisor of the numbers. Thus, the numbers 30, $(2 \times 3 \times 5)$, and 40, $(2 \times 2 \times 2 \times 5)$, have the factors 2 and 5 in common. So the common divisors of 30 and 40 are 2, 5, and 10, and the greatest common divisor is 10.

Rule.—*To find the greatest common divisor of two or more numbers, find the product of the prime factors common to the numbers.*

2. Find the g. c. d. of 50, 75, and 125.*

Operation No. 1.

$50 = 2 \times 5 \times 5.$

$75 = 3 \times 5 \times 5.$

$125 = 5 \times 5 \times 5.$

$5 \times 5 = 25$, g. c. d.

Operation No. 2.

5	50	75	125.
5	10	15	25.
	2	3	5

$5 \times 5 = 25$, g. c. d.

3. Find the g. c. d. of 80, 100, 140.

4. Find the g. c. d. of 48, 60, 72.

5. Find the g. c. d. of 64, 96, 256.

*The letters g. c. d. are sometimes used instead of the words *greatest common divisor.*

Divisibility of Numbers.

6. Find the g. c. d. of 640 and 760.

Operation.

640)760(1
640
120)640(5
600
40)120(3
120

Explanation.

The number 760 is an integral number of times the g. c. d., *whatever that may be;* so is the number 640. We make an incomplete division of 760 by 640 and have as a remainder the number 120. Since 640 and 760 are each an integral number of times the g. c. d., their difference, 120, must be an integral number of times the g. c. d.: for, taking an integral number of times a thing from an integral number of times a thing, must leave an integral number of times the thing. Therefore, no number greater than 120 can be the g. c. d. But if 120 is an exact divisor of 640, it is also an exact divisor of 760, for it will be contained one more time in 760 than in 640. We make the trial and find that 120 is not an exact divisor of 640; there is a remainder of 40. Since 600, (120 × 5), and 640 are each an integral number of times the g. c. d., 40 must be an integral number of times the g. c. d. But if 40 is an exact divisor of 120 it is an exact divisor of 600, (120 × 5), and 640, (40 more than 600), and 760, (120 more than 640). We make the trial and find that it is an exact divisor of 120, and is therefore the g. c. d. of 640 and 760.

From the foregoing learn that any number that is an exact divisor of two numbers is an exact divisor of their difference.

169. From the foregoing make a rule for finding the g. c. d. of two numbers and apply it to the following

PROBLEMS.

Find the g. c. d.

1. Of 380 and 240.
2. Of 275 and 155.
3. Of 144 and 96.
4. Of 1728 and 288.
5. Of 650 and 175.
6. Of 540 and 450.
7. Of 320 and 860.
8. Of 475 and 350.
9. Of 390 and 520.
10. Of 450 and 600.

(a) Find the sum of the ten results.

Algebra—Equations.

170. An equation is the expression of the equality of two numbers or combinations of numbers.

EQUATIONS.

(1) $2 + 4 + 6 = 3 + 5 + 4$

(2) $a + b + c = 40 - 12$

(3) $2x + x + 4 = 10 + 9$

1. Every equation is made up of two members. The part of the equation which is on the left of the sign of equality is called the first member; the part on the right of the sign of equality, the second member.

2. The first member of equation No. 1 (above) is ——— It is made up of ——— terms.

3. If the same number be added to each member of an equation, the equality will not be destroyed.

If $x = 8$, then $x + 4 = 8 + 4$.

If $a + b = 16$, then $a + b + c = 16 + c$.

4. If the same number be subtracted from each member of an equation, the equality will not be destroyed.

If $x = 8$, then $x - 3 = 8 - 3$.

If $a + b = 16$, then $a + b - c = 16 - c$.

5. If each member of an equation be multiplied by the same number, the equality will not be destroyed.

If $x = 8$, then $4x = 4$ times 8, or 32.

If $a + b = 16$, then $4a + 4b = 4$ times 16, or 64.

6. If each member of an equation be divided by the same number, the equality will not be destroyed.

If $x = 8$, then $\frac{x}{4} = \frac{8}{4}$, or 2.

If $a + b = 16$, then $\frac{a}{4} + \frac{b}{4} = \frac{16}{4}$, or 4.

Algebra—Equations.

7. Any term in an equation may be transposed from one member of the equation to the other; but its sign must be changed when the transposition is made.

If $x + 5 = 15$, then $x = 15 - 5$, or 10.*
If $y - 6 = 27$, then $y = 27 + 6$, or 33.†
If $a + b + c = 18$, then $a + b = 18 - c$.
If $x + y - z = 25$, then $x + y = 25 + z$.

171. To find the number for which x stands, in an equation in which there is no other unknown number.

EXAMPLE NO. 1.

Equation,	$x + 2x + 3x - 5 = 13$
Transposing,	$x + 2x + 3x = 13 + 5$
Uniting,	$6x = 18$
Dividing,	$x = 3$

EXAMPLE NO. 2.

Equation,	$2x + 3x + 6 = 5x - 2x + 18$
Transposing,	$2x + 3x - 5x + 2x = 18 - 6$
Uniting,	$2x = 12$
Dividing,	$x = 6$

PROBLEMS.

Find the value of x.

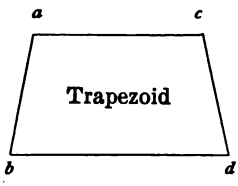
1. $x + 4 = 12$
2. $x + 3x = 8$
3. $5x - 2 = 23$
4. $3x - x = 44$
5. $7x + x = 144$.
6. $3x + 2x - 4 = x + 16$
7. $5x - 7 = 3x + 5$
8. $7x + 2x - x = 3x + 35$
9. $5x - 4x - 3x + 6x = 44$
10. $6x - 8 - 2x = 3x + 5$

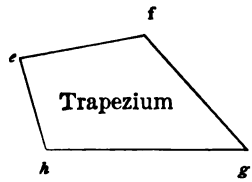
(*a*) Find the sum of the ten results.

*Observe that 5 is subtracted from each member of the equation.
†Observe that 6 is added to each member of the equation.

Geometry.

172. Quadrilaterals that are not parallelograms.





1. Two of the sides of a trapezoid are parallel and two are not parallel. In the trapezoid represented above the side *ac* is parallel to the side ————.

2. No two of the bounding lines of a trapezium are parallel.

3. In the trapezoid represented above no one of the angles is a right angle. Name the angles that are greater than right angles; the angles that are less than right angles.

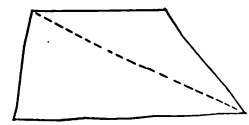
4. Draw a trapezoid two of whose angles are right angles.

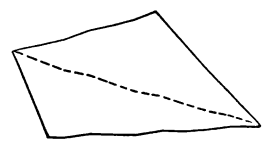
5. Can you draw a trapezoid having one and only one right angle?

6. Draw a trapezium one of whose angles is a right angle.

7. Can you draw a trapezium having more than one right angle?

8. Every quadrilateral (trapezium, trapezoid, or parallelogram) may be divided into two triangles. Remember that the sum of the angles of two triangles is equal to four right angles. Observe that the sum of the angles of the two triangles is equal to the sum of the angles of the quadrilateral. *So the sum of the angles of a quadrilateral is equal to four right angles.*





173. Miscellaneous Review.

1. If two of the angles of a trapezoid are right angles and the third is an angle of 60°, how many degrees in the fourth angle? Draw such a trapezoid.*

2. If the sum of three of the angles of a trapezium is 298°, how many degrees in the fourth angle? Draw such a trapezium.*

3. If one of the angles of a triangle is an angle of 80°, and the other two angles are equal, how many degrees in each of the other angles? Draw the figure.*

4. If one of the angles of a quadrilateral is a right angle, and the other three angles are equal, what kind of a quadrilateral is the figure?

5. One of the angles of a quadrilateral is a degrees; another is b degrees; the third is c degrees. How many degrees in the fourth angle‡

6. The smallest angle of a triangle is x degrees; another angle is $2x$ degrees, and the third is $3x$ degrees:

Then $x + 2x + 3x = 180$.

Find the value of x; of $2x$; of $3x$.

7. 643,265,245,350. Without performing the division tell whether this number is exactly divisible by 9; by 5; by 10; by 25; by 50; by $12\frac{1}{2}$; by 18; by 6; by 15; by 30; by 90; by $16\frac{2}{3}$.†

8. A number is made up of the following prime factors: 2, 2, 3, 3, 5, 7, 11. Is the number exactly divisible by 18? by 26? by 35? by 77? by 21? by 30? by 45? by 8?

* It is not expected that this drawing will be accurate in its angular measurements—simply an approximation to accuracy, to aid the pupil in recognizing the comparative size of angles.

† A careful study of pages 71-75 inclusive will enable the pupil to make the statements called for, with little hesitation.

‡ See problem 3, page 70.

FRACTIONS.

174. A fraction may be expressed by two numbers, one of them being written above and the other below a short horizontal line; thus, $\frac{3}{5}$, $\frac{11}{17}$, $\frac{249}{375}$.

175. The number above the line is the **numerator** of the fraction; the number below the line, the **denominator** of the fraction.

176. Kinds of Fractions.

1. A fraction whose numerator is less than its denominator is a **proper fraction.**

$\frac{3}{4}$, $\frac{2}{5}$, $\frac{18}{17}$, are proper fractions.

2. A fraction whose numerator is equal to or greater than its denominator is an **improper fraction.**

$\frac{5}{3}$, $\frac{6}{6}$, $\frac{27}{4}$, are improper fractions.

Note.—The fraction .7 is a proper fraction. 2.7 may be regarded as an improper fraction or as a mixed number. If it is to be considered an improper fraction it should be read, 27 tenths; if a mixed number, 2 and 7 tenths.

3. Such expressions as the following are **compound fractions:**

$\frac{3}{4}$ of $\frac{6}{7}$, $\frac{2}{5}$ of $\frac{1}{2}$, $\frac{5}{8}$ of $\frac{7}{11}$.

4. A fraction whose numerator or denominator is itself a fraction or a mixed number, is a **complex fraction.**

$\frac{\frac{3}{4}}{4}$, $\frac{2}{3\frac{1}{2}}$, $\frac{\frac{1}{2}}{2\frac{1}{4}}$, are complex fractions.

Fractions.

5. Any fraction that is neither compound nor complex is a **simple fraction.**

$\frac{3}{5}$, $1\frac{2}{11}$, $\frac{5}{3}\frac{4}{7}$, are simple fractions.

6. A fraction whose denominator is 1 with one or more zeros annexed to it, is a **decimal fraction.**

$\frac{3}{10}$, .7, .25, $\frac{3}{1}\frac{6}{0}\frac{}{0}$, are decimal fractions.

NOTE 1.—The denominator of a decimal fraction may be expressed by figures or it may be indicated by the position of the right-hand figure of its numerator with reference to the decimal point. When the denominator is thus indicated, the fraction is called a **decimal** and is said to be *written decimally.*

NOTE 2.—All fractions that are not decimal are called **common fractions.** A decimal fraction when not "written decimally" (or thought of as written decimally) is usually classed as a common fraction.

7. A **complex decimal** is a decimal and a common fraction combined in one number.

$.7\frac{1}{8}$, $.25\frac{1}{7}$, $.056\frac{2}{3}$, are complex decimals.

177. There are **three aspects** in which fractions should now be considered.

1. THE FRACTIONAL UNIT ASPECT.

The numerator tells the *number of things* and the denominator *indicates their name.* In the fraction $\frac{5}{7}$ there are 5 things (magnitudes) called sevenths. In the fraction $\frac{5}{8}$ there are *five fractional units* each of which is one eighth of some other unit called the *unit of the fraction.*

NOTE.—The function of the denominator is to show the number of parts into which the unit of the fraction is divided; the function of the numerator, to show the number of parts (fractional units) taken.

Fractions.

II. THE DIVISION ASPECT.

The numerator of a fraction is *a dividend;* the denominator, *a divisor*, and the fraction itself, *a quotient:* thus, in the fraction $\frac{5}{8}$, the dividend is 5; the divisor, 8, and the quotient, $\frac{5}{8}$.

NOTE.—In the case of an improper fraction, as $\frac{8}{4}$, it may be more readily seen by the pupil that the numerator is the dividend, the denominator the divisor, and the fraction ($\frac{8}{4}=2$) the quotient; but the division relation is in every fraction, whether proper or improper, common or decimal, simple or complex.

III. THE RATIO ASPECT.

The numerator of a fraction is *an antecedent;* the denominator, *a consequent*, and the fraction itself, *a ratio:* thus, in the fraction $\frac{7}{10}$, 7 is the antecedent, 10 the consequent, and $\frac{7}{10}$ the ratio.

NOTE 1.—This relation may be more readily seen by the pupil in the case of an improper fraction. In the fraction $\frac{12}{4}$, 12 is the antecedent; 4, the consequent; $\frac{12}{4}$, or 3, the ratio.

NOTE 2.—*Every integral number as well as every fraction is a ratio.* The number 8 is the ratio of a magnitude that is 8 times some unit of measurement to a magnitude that is 1 time the same unit of measurement. 8 (units of measurement) is the antecedent; 1 (unit of measurement) is the consequent, and the pure number 8 is the ratio.

178. REDUCTION OF FRACTIONS.

1. The numerator and the denominator of a fraction are its **terms.**

2. A fraction is said to be in its **lowest terms** when its numerator and denominator are integral numbers that are prime to each other.

Fractions.

3. Reduce $\frac{160}{200}$ to its lowest terms.

Operation.

$10)\frac{160}{200} = \frac{16}{20}$.

$4)\frac{16}{20} = \frac{4}{5}$.

Explanation.

Dividing each term of $\frac{160}{200}$ by 10 we have 1 tenth as many parts, which are 10 times as large. Dividing each term of $\frac{16}{20}$ by 4 we have 1 fourth as many parts, which are 4 times as large. Hence, $\frac{160}{200} = \frac{4}{5}$. But 4 and 5 are prime to each other, and the fraction is in its lowest terms.

RULE.—*Divide each term of the fraction by any common divisor except 1, and divide each term of the fraction thus obtained by any common divisor except 1, and so continue until the terms are prime to each other.*

Reduce to lowest terms.

(1) $\frac{275}{375}$ (2) $\frac{520}{650}$ (3) $\frac{156}{270}$ (4) $\frac{235}{340}$

(5) $\frac{56}{210}$ (6) $\frac{36}{180}$ (7) $\frac{171}{405}$ (8) $\frac{63}{204}$

(9) $\frac{37\frac{1}{2}}{100}$* (10) $\frac{2\frac{1}{2}}{4}$†

(a) Find the sum of the ten results.‡

4. Reduce $\frac{5}{8}$ to higher terms — to 120ths.

Operation.

$120 \div 8 = 15$.

$\frac{5 \times 15}{8 \times 15} = \frac{75}{120}$

Explanation.

In $\frac{75}{120}$ there are 15 times as many parts as there are in $\frac{5}{8}$, and the parts are 1 fifteenth as large. Hence, $\frac{75}{120} = \frac{5}{8}$.

* Divide each term by $12\frac{1}{2}$. This involves the reduction of a complex to a simple fraction; but it will lead to thoughtful work for the pupil to solve such problems in this manner.

† Divide each term by $\frac{1}{2}$.

‡ If the pupil has not had sufficient practice in addition of fractions to do this, the finding of the sum may be omitted until the book is reviewed.

Fractions.

Reduce to higher terms — to 160ths.

(1) $\frac{5}{8}$ (2) $\frac{11}{16}$ (3) $\frac{9}{80}$ (4) $\frac{37}{40}$
(5) $\frac{3}{40}$ (6) $\frac{3}{8}$ (7) $\frac{5}{16}$ (8) $\frac{11}{80}$
(9) $\frac{37}{80}$ (10) $\frac{43}{80}$

(a) Find the sum of the ten results.

5. Two or more fractions whose denominators are the same, are said to have **a common denominator.**

6. Two or more fractions that do not have a common denominator may be changed to equivalent fractions having a common denominator.

EXAMPLE.

$\frac{2}{3}$ and $\frac{3}{4}$ may be changed to 12ths, 24ths, or 36ths.

$\frac{2}{3} = \frac{8}{12}$ $\frac{3}{4} = \frac{9}{12}$ $\frac{2}{3} = \frac{16}{24}$ $\frac{3}{4} = \frac{18}{24}$

7. Two or more fractions that do not have a common denominator may be changed to equivalent fractions having their least common denominator. The l. c. d. of two or more fractions is the l. c. m. of the given denominators.

EXAMPLE.

Change $\frac{11}{30}$, $\frac{9}{40}$, and $\frac{37}{60}$ to equivalent fractions having their least common denominator.

OPERATION.

(1) The l. c. m. of 30, 40, and 60 is 120.

(2) $120 \div 30 = 4$ $\frac{11 \times 4}{30 \times 4} = \frac{44}{120}$

(3) $120 \div 40 = 3$ $\frac{9 \times 3}{40 \times 3} = \frac{27}{120}$

(4) $120 \div 60 = 2$ $\frac{37 \times 2}{60 \times 2} = \frac{74}{120}$

$\frac{11}{30} = \frac{44}{120}$, $\frac{9}{40} = \frac{27}{120}$, $\frac{37}{60} = \frac{74}{120}$

Fractions.

Reduce to equivalent fractions having their l. c. d.

(1) $\frac{11}{15}$ and $\frac{9}{20}$
(2) $\frac{19}{25}$ and $\frac{13}{20}$
(3) $\frac{23}{30}$ and $\frac{17}{40}$
(4) $\frac{7}{36}$ and $\frac{5}{63}$
(5) $\frac{8}{45}$ and $\frac{37}{60}$
(6) $\frac{4}{15}$, $\frac{2}{5}$, and $\frac{11}{20}$
(7) $\frac{6}{25}$, $\frac{1}{5}$, and $\frac{7}{20}$
(8) $\frac{7}{30}$, $\frac{2}{5}$, and $\frac{23}{40}$
(9) $\frac{28}{36}$, $\frac{7}{10}$, and $\frac{58}{63}$
(10) $\frac{37}{45}$, $\frac{3}{10}$, and $\frac{23}{60}$

(a) Find the sum of the twenty-five fractions.*

179. To add common fractions.

RULE.—*Reduce the fractions if necessary to equivalent fractions having a common denominator, add their numerators, and write their sum over the common denominator.*

EXAMPLE.

Add $\frac{11}{45}$, $\frac{17}{30}$, and $\frac{53}{60}$.

(1) The l. c. m. of 45, 30, and 60, is 180.
(2) $\frac{11}{45} = \frac{44}{180}$. $\frac{17}{30} = \frac{102}{180}$. $\frac{53}{60} = \frac{159}{180}$.
(3) $\frac{44}{180} + \frac{102}{180} + \frac{159}{180} = \frac{305}{180}$.

NOTE.—If the work that precedes this article has been well done, no explanation of the foregoing will be necessary. Pupils have already learned (presumably before using this book) (1) that fractions may be reduced to higher terms, (2) that two or more fractions whose denominators are not alike may be reduced to higher terms with like denominators, (3) that a common denominator of two or more fractions with unlike denominators, is a common multiple of the given denominators, and (4) that in reducing a fraction to higher terms the numerator and denominator must be multiplied by the same number. The simple problem of adding 44 180ths, 102 180ths, and 159 180ths, is not unlike the problem of adding 44 apples, 102 apples, and 159 apples.

(For a continuation of this work, see page 91.)

* This work may be omitted until the subject of fractions is reviewed.

Algebraic Fractions.

180. The expressions $\frac{a}{b}$, $\frac{x}{4}$, $\frac{6}{cd}$, are algebraic fractions.

The above expressions are read, a divided by b; x divided by 4; 6 divided by cd.

REDUCTION OF ALGEBRAIC FRACTIONS.

181. Reduce to lowest terms:

1. $\frac{ab}{a^3} = \left(\frac{a \times b}{a \times a \times a}\right)$. $\frac{ab \div a}{a^3 \div a} = \frac{b}{a^2}$

2. $\frac{4a}{6b} = \left(\frac{2 \times 2 \times a}{2 \times 3 \times b}\right)$. $\frac{4a \div 2}{6b \div 2} = \frac{2a}{3b}$

3. $\frac{abc}{bcd} = \left(\frac{a \times b \times c}{b \times c \times d}\right)$. $\frac{abc \div bc}{bcd \div bc} = \frac{a}{d}$

Let $a = 2$, $b = 3$, $c = 5$, and $d = 7$, and verify.

Observe that to reduce a fraction to its lowest terms we have only to strike out the factors that are common to its numerator and denominator.

4. $\frac{a^2b}{a^3c}$. What factors are common to both numerator and denominator? Reduce and verify.

5. $\frac{x^2y^3}{y^2z^2}$. What factors are common to both numerator and denominator? Reduce and verify.

6. $\frac{a^2b^3}{a^5b^5} =$ 7. $\frac{xy^2}{x^3y^5} =$ 8. $\frac{4a + 4b}{6c + 8d} =$

9. $\frac{abc}{ax} =$ 10. $\frac{2ax}{4a^2x} =$ 11. $\frac{3x + 6y}{12} =$

Algebraic Fractions.

182. Reduce to Higher Terms.

1. Change $\frac{2a}{bc}$ to a fraction whose denominator is abc.

$\frac{2a \times a}{bc \times a} = \frac{2a^2}{abc}$ Let $a = 2$, $b = 3$, and $c = 5$, and verify the reduction.

2. Change $\frac{3x}{2ay}$ to a fraction whose denominator is $2ay^2$.

$\frac{3x \times y}{2ay \times y} = \frac{3xy}{2ay^2}$ Give any values you please to a, x, and y and verify the reduction.

183. Reduce to Equivalent Fractions Having a Common Denominator.

1. $\frac{x}{ab}$ and $\frac{y}{a^2d}$ Since the common denominator must be exactly divisible by each of the given denominators, it must contain all the prime factors* found in either of the given denominators. The new denominator must therefore be $a \times a \times b \times d = a^2bd$; $a^2bd \div ab = ad$; $a^2bd \div a^2d = b$.

$$\frac{x \times ad}{ab \times ad} = \frac{adx}{a^2bd}. \qquad \frac{y \times b}{a^2d \times b} = \frac{by}{a^2bd}$$

Give any values you please to a, b, d, x, and y, and verify.

2. $\frac{4}{ab^2}$ and $\frac{3}{bc^2}$ The common denominator must contain the factors a, b, b, c, c. Reduce and verify.

3. $\frac{xy}{5}$ and $\frac{yz}{a}$ The common denominator is $5a$. Reduce and verify.

* Since the numerical values of the letters are unknown, each must be regarded as prime to all the others. The prime factors, then, in the first denominator are a and b; in the second, a, a, and d.

GEOMETRY.

184. QUADRILATERALS.—REVIEW.

1. All the geometrical figures on this page are quadrilaterals; that is, *each has four sides.*

2. The first four figures are parallelograms; that is, *the opposite sides of each figure are parallel.*

3. The first two figures are rectangular; that is, *their angles are right angles.*

4. The first and third are equilateral; that is, *the sides are equal.*

5. There is one equilateral rectangular parallelogram. Which is it?

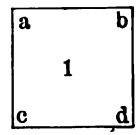
6. There is one equilateral parallelogram that is not rectangular. Which is it?

7. There is one rectangular parallelogram that is not equilateral. Which is it?

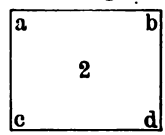
8. The sum of the angles of each figure on the page is ..... right angles.

9. Tell as nearly as you can the size of each angle of each figure.

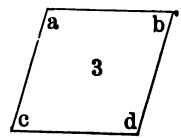
Square.



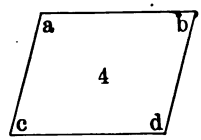
Oblong.



Rhombus.



Rhomboid.



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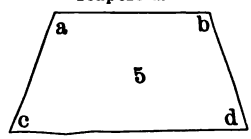

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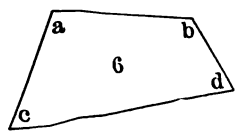


185. Miscellaneous Review.

1. The difference of two numbers is $374\frac{1}{10}$; the smaller number is $243\frac{7}{30}$. What is the larger number? *

2. The difference of two numbers is a; the smaller number is b. What is the larger number?

3. James had a certain number of dollars and John had three times as many; together they had 196 dollars. How many had each? $(x + 3x = 196)$ †

4. William had a certain number of marbles; Henry had twice as many as William, and George had twice as many as Henry; together they had 161. How many had each? $(x + 2x + 4x = 161)$†

5. Divide 140 dollars between two men, giving to one man 30 dollars more than to the other. $(x + x + 30 = 140)$†

6. By what integral numbers is 30, $(2 \times 3 \times 5)$, exactly divisible besides itself and 1?

7. By what is abc, $(a \times b \times c)$, exactly divisible besides itself and 1?

(1) How many times is a contained in abc?
(2) How many times is b contained in abc?
(3) How many times is c contained in abc?
(4) How many times is ab contained in abc?
(5) How many times is ac contained in abc?
(6) How many times is bc contained in abc?

Observe that a number composed of three different prime factors has —— exact integral divisors.

8. Change $\frac{5}{8}$ to 60ths. Is $\frac{5}{8}$ more or less than $\frac{37}{60}$?
9. Change $\frac{3}{8}$ to 100ths. Change $\frac{2}{3}$ to 100ths.
10. Change $\frac{1}{6}$ to 100ths. Change $\frac{5}{6}$ to 100ths.

* The difference of two numbers is 5; the smaller number is 4. What is the larger number?

† See page 78.

Art. 179, continued from page 86.

Find the sum of—

1. $\frac{5}{16}$ and $\frac{7}{20}$
2. $\frac{4}{15}$ and $\frac{9}{18}$
3. $\frac{11}{21}$ and $\frac{5}{28}$
4. $\frac{7}{15}$ and $\frac{13}{45}$
5. $\frac{5}{14}$ and $\frac{7}{10}$
6. $\frac{11}{16}$, $\frac{1}{2}$, and $\frac{13}{20}$
7. $\frac{11}{15}$, $\frac{1}{3}$, and $\frac{9}{18}$
8. $\frac{10}{21}$, $\frac{1}{4}$, and $\frac{23}{28}$
9. $\frac{8}{15}$, $\frac{2}{3}$, and $\frac{32}{45}$
10. $\frac{9}{14}$, $\frac{1}{4}$, and $\frac{3}{10}$

(a) Find the sum of the ten sums.

186. To Subtract Common Fractions.

Rule.—*Reduce the fractions if necessary to equivalent fractions having a common denominator, find the difference of their numerators, and write it over the common denominator.*

Example.

From $\frac{11}{25}$ subtract $\frac{7}{35}$.

(1) The l. c. m. of 25 and 35 is 175.

(2) $\frac{11}{25} = \frac{77}{175}$ $\frac{7}{35} = \frac{35}{175}$

(3) $\frac{77}{175} - \frac{35}{175} = \frac{42}{175}$

Compare the following:

77 175ths — 35 175ths = 42 175ths.
77 apples — 35 apples = 42 apples.

Find the difference of—

1. $\frac{3}{5}$ and $\frac{5}{18}$
2. $\frac{2}{5}$ and $\frac{7}{18}$
3. $\frac{9}{20}$ and $\frac{1}{3}$
4. $\frac{11}{20}$ and $\frac{1}{6}$
5. $\frac{5}{6} - \frac{1}{2}$
6. $\frac{4}{9} - \frac{1}{3}$

(b) Find the sum of the six differences.

Fractions.

187. *To subtract one mixed number from another when the fraction in the subtrahend is greater than the fraction in the minuend.*

EXAMPLE.

From $58\frac{3}{8}$ take $32\frac{5}{8}$.

Operation.	*Explanation.*
$58\frac{3}{8} = 58\frac{9}{24}$	$\frac{5}{8}$ is greater than $\frac{9}{24}$, therefore we take 1 unit from the 8 units, change it to 24ths, and add it to the 9 24ths.
$32\frac{5}{8} = 32\frac{15}{24}$	
Difference $25\frac{17}{24}$	$\frac{3}{8} + \frac{9}{24} = \frac{33}{24}$. $\frac{33}{24} - \frac{15}{24} = \frac{17}{24}$.

2 units from 7, (8 — 1), units = 5 units. 3 tens from 5 tens = 2 tens.

I. Find the difference of—

1. $24\frac{2}{7}$ and $16\frac{2}{5}$
2. $29\frac{3}{5}$ and $15\frac{1}{4}$
3. $46\frac{19}{56}$ and $18\frac{7}{20}$
4. $52\frac{5}{8}$ and $31\frac{3}{5}$
5. $47\frac{3}{5}$ and $18\frac{2}{5}$
6. $35\frac{1}{4}$ and $26\frac{1}{2}$
7. $28\frac{3}{7}$ and $14\frac{3}{5}$
8. $36\frac{9}{28}$ and $8\frac{9}{10}$
9. $65\frac{1}{2}$ and $22\frac{3}{4}$
10. $34\frac{1}{3}$ and $27\frac{1}{4}$

(a) Find the sum of the ten differences.

II. Reduce to simplest form—

1. $5\frac{1}{2} + 3\frac{1}{3} - 5\frac{1}{4}$
2. $6\frac{2}{5} - 3\frac{1}{3} + 4\frac{1}{5}$
3. $2\frac{1}{5} - 1\frac{1}{4} + 3\frac{2}{5}$
4. $7\frac{3}{4} + 3\frac{3}{8} - 1\frac{3}{5}$
5. $6\frac{1}{2} - 3\frac{1}{4} + 5\frac{1}{5}$
6. $3\frac{1}{4} + 4\frac{1}{2} - 3\frac{1}{8} - 2\frac{3}{8} + 1\frac{3}{4}$
7. $6\frac{3}{5} - 2\frac{1}{4} - 1\frac{3}{4} - 1\frac{21}{40} + 2\frac{1}{4}$
8. $5\frac{1}{2} + 4\frac{3}{5} + 2\frac{1}{8} + 3\frac{7}{8} + 3\frac{1}{4}$

(b) Find the sum of the eight results.

Fractions.

188. To multiply a fraction by an integer.

Multiply $\frac{7}{24}$ by 6.

Operation No. 1.	Operation No. 2.
6 times $\frac{7}{24}$ are $\frac{42}{24} = 1\frac{3}{4}$	6 times $\frac{7}{24} = \frac{7}{4} = 1\frac{3}{4}$

1. *Observe* that by the first operation we obtain $\frac{42}{24}$; that in $\frac{42}{24}$ there are 6 times as many parts as there are in $\frac{7}{24}$ and that the parts are of the same size as those in $\frac{7}{24}$.

2. *Observe* that by the second operation we obtain $\frac{7}{4}$; that in $\frac{7}{4}$ there are the same number of parts as there are in $\frac{7}{24}$, and that the parts are 6 times as great as those in $\frac{7}{24}$.

Note 1.—The 7 of $\frac{7}{24}$ may be regarded as a dividend; the 24, as a divisor, and $\frac{7}{24}$ itself, as a quotient. In $\frac{42}{24}$, we have a dividend 6 times as great as that in $\frac{7}{24}$, the divisor remainiug unchanged. In $\frac{7}{4}$, we have a divisor 1 sixth as great as that in $\frac{7}{24}$, the dividend remaining unchanged. Multiplying the dividend or dividing the divisor by any number, multiplies the quotient by the same number.

Note 2.—The 7 of $\frac{7}{24}$ may be regarded as the antecedent of a couplet; the 24, as the consequent, and $\frac{7}{24}$ itself as the ratio. Multiplying the antecedent or dividing the consequent of any couplet multiplies the ratio by the same number.

RULE.—*To multiply a fraction by an integer, multiply its numerator or divide its denominator by the integer.*

I. Find the product.

1. $\frac{5}{12} \times 4$	5. $\frac{5}{4} \times 8$	9. $\frac{7}{12} \times 4$
2. $\frac{8}{15} \times 6$	6. $\frac{3}{5} \times 9$	10. $\frac{7}{15} \times 6$
3. $11\frac{1}{20} \times 5$	7. $\frac{3}{4} \times 8$	11. $\frac{9}{20} \times 5$
4. $17\frac{1}{30} \times 7$	8. $\frac{2}{5} \times 9$	12. $13\frac{1}{30} \times 7$

(a) Find the sum of the twelve products.

II. Find the product.

1. $3\frac{1}{2} \times 7$	5. $4\frac{3}{5} \times 5$	9. 6.3×5
2. $5\frac{3}{8} \times 6$	6. $1\frac{7}{8} \times 4$	10. $21\frac{3}{20} \times 4$
3. $7\frac{7}{20} \times 4$	7. $5\frac{3}{5} \times 5$	11. $4\frac{5}{8} \times 6$
4. 3.7×5	8. $8\frac{1}{8} \times 4$	12. $6\frac{1}{2} \times 7$

(b) Find the sum of the twelve products.

Fractions.

189. To divide a fraction by an integer.

Divide $\frac{6}{7}$ by 3.

Operation No. 1.	Operation No. 2.
One third of $\frac{6}{7} = \frac{2}{7}$	One third of $\frac{1}{7} = \frac{1}{21}$
	One third of $\frac{6}{7} = \frac{6}{21} = \frac{2}{7}$

1. *Observe* that by the first operation we obtain $\frac{2}{7}$; that in $\frac{2}{7}$ there are 1 third as many parts as there are in $\frac{6}{7}$ and that the parts are of the same size as those in $\frac{6}{7}$.

2. *Observe* that by the second operation we obtain $\frac{6}{21}$; that in $\frac{6}{21}$ there are the same number of parts as there are in $\frac{6}{7}$ and that the parts are 1 third as great as those in $\frac{6}{7}$.

Note 1.—The 6 of $\frac{6}{7}$ may be regarded as a dividend; the 7 as a divisor, and the $\frac{6}{7}$ itself as a quotient. In $\frac{2}{7}$, we have a dividend 1 third as great as that in $\frac{6}{7}$, the divisor remaining unchanged. In $\frac{6}{21}$, we have a divisor 3 times as great as that in $\frac{6}{7}$, the dividend remaining unchanged. Dividing the dividend or multiplying the divisor by any number, divides the quotient by the same number.

RULE.—*To divide a fraction by an integer, divide its numerator or multiply its denominator by the integer.*

I. Find the quotient. (See p. 105, problems 15 and 16.)

1. $\frac{7}{18} \div 4$	4. $\frac{5}{6} \div 4$	7. $1\frac{7}{24} \div 4$
2. $\frac{7}{18} \div 5$	5. $\frac{5}{6} \div 5$	8. $1\frac{7}{24} \div 5$
3. $\frac{7}{18} \div 20$	6. $\frac{5}{6} \div 20$	9. $1\frac{7}{24} \div 20$

(a) Find the sum of the nine quotients.

II. Find the quotient. (See p. 105, problems 17 and 18.)

1. $17\frac{1}{5} \div 3$	4. $18\frac{3}{10} \div 3$	7. $16\frac{1}{2} \div 3$
2. $17\frac{1}{5} \div 4$	5. $18\frac{3}{10} \div 4$	8. $16\frac{1}{2} \div 4$
3. $17\frac{1}{5} \div 6$	6. $18\frac{3}{10} \div 6$	9. $16\frac{1}{2} \div 6$

(b) Find the sum of the nine quotients.

Fractions.

190. To Multiply by a Fraction.

$6 multiplied by 3, means, take 3 times $6. $6 × 3 = $18.

$6 multiplied by 2, means, take 2 times $6. $6 × 2 = $12.

$6 multiplied by $2\frac{1}{2}$, means, take $2\frac{1}{2}$ times $6; or 2 times $6 + $\frac{1}{2}$ of $6. $6 × $2\frac{1}{2}$ = $15.

$6 multiplied by $\frac{1}{2}$, means, take $\frac{1}{2}$ of $6. $6 × $\frac{1}{2}$ = $3.

$6 multiplied by $\frac{2}{3}$, means, take $\frac{2}{3}$ of $6. $6 × $\frac{2}{3}$ = $4.

To the Teacher.—Require the pupil to examine the preceding statements and similar ones presented by the teacher or by himself, until he clearly understands that to multiply by a fraction is to take such part of the multiplicand as is indicated by the fraction. Thus: to multiply 48 by $\frac{3}{4}$ is to take three fourths of 48; that is, *three times 1 fourth of 48.* It will thus be clear that multiplication by a fraction involves both multiplication and division; hence the work on the preceding pages should be mastered by the pupil before attempting what follows.

Example I.

Multiply 24 by $\frac{3}{4}$.
1 fourth of 24 is 6.
3 fourths of 24 are 18.

Example II.

Multiply $\frac{3}{5}$ by $\frac{3}{4}$.
1 fourth of $\frac{3}{5}$ is $\frac{3}{20}$.
3 fourths of $\frac{3}{5}$ are $\frac{9}{20}$.

Example III.

Multiply $275\frac{3}{5}$ by $\frac{3}{4}$.
1 fourth of $275\frac{3}{5}$ is $68\frac{9}{20}$.
3 fourths of $275\frac{3}{5}$ are $206\frac{7}{20}$.

Example IV.

Multiply $346\frac{2}{5}$ by $2\frac{1}{2}$.*
Two times $346\frac{2}{5}$ = $692\frac{4}{5}$.
1 half of $346\frac{2}{5}$ = $173\frac{1}{5}$.
$692\frac{4}{5}$ + $173\frac{1}{5}$ = 866 Ans.

Rule.—*To multiply by a fraction, divide the multiplicand by the denominator of the fraction and multiply the quotient thus obtained by the numerator of the fraction.*

Observe that in practice we may, if more convenient, multiply the multiplicand by the numerator of the fraction, and divide the product thus obtained by the denominator. To multiply 12 by $\frac{3}{4}$ we may take 3 times 1 fourth of 12 or 1 fourth of 3 times 12, as we choose

*This means, take 2 times $346\frac{2}{5}$ and $\frac{1}{2}$ of $346\frac{2}{5}$.

Fractions.

I. Find the product. (See p. 105, prob. 19 and 20.)

1. $345 \times \frac{1}{2}$	4. $263 \times \frac{1}{3}$	7. $263 \times \frac{1}{4}$
2. $345 \times \frac{3}{10}$*	5. $263 \times \frac{2}{4}$	8. $576 \times \frac{3}{8}$
3. $345 \times \frac{1}{5}$	6. $263 \times \frac{2}{5}$	9. $576 \times \frac{3}{4}$

(a) Find the sum of the nine products.

II. Find the product. (See p. 105, prob. 21 and 22.)

1. $\frac{7}{16} \times \frac{1}{2}$	4. $\frac{5}{8} \times \frac{1}{3}$	7. $\frac{5}{8} \times \frac{1}{4}$
2. $\frac{7}{16} \times \frac{3}{10}$†	5. $\frac{5}{8} \times \frac{3}{4}$	8. $\frac{1}{2} \times \frac{3}{8}$
3. $\frac{7}{16} \times \frac{1}{5}$	6. $\frac{5}{8} \times \frac{2}{5}$	9. $\frac{1}{2} \times \frac{3}{4}$

(b) Find the sum of the nine products.

III. Find the product. (See p. 106, prob. 23 and 24.)

1. $372\frac{1}{4} \times \frac{1}{2}$	4. $523\frac{2}{3} \times \frac{1}{3}$	7. $523\frac{2}{3} \times \frac{1}{4}$
2. $372\frac{1}{4} \times \frac{3}{10}$	5. $523\frac{2}{3} \times \frac{3}{4}$	8. $153\frac{1}{2} \times \frac{3}{8}$
3. $372\frac{1}{4} \times \frac{1}{5}$	6. $523\frac{2}{3} \times \frac{2}{3}$	9. $153\frac{1}{2} \times \frac{3}{4}$

(c) Find the sum of the nine products.

IV. Find the product. (See p. 106, prob. 25 and 26.)

1. $462\frac{2}{3} \times 2\frac{1}{10}$	6. $346\frac{1}{6} \times 3\frac{2}{3}$
2. $462\frac{2}{3} \times 3\frac{7}{10}$	7. $346\frac{1}{6} \times 2\frac{1}{4}$
3. $462\frac{2}{3} \times 2\frac{1}{5}$	8. $275\frac{1}{2} \times 4\frac{1}{3}$
4. $346\frac{1}{6} \times 2\frac{1}{5}$	9. $275\frac{1}{3} \times 3\frac{1}{4}$
5. $346\frac{1}{6} \times 3\frac{3}{4}$	10. $275\frac{1}{2} \times 2\frac{1}{4}$

(d) Find the sum of the ten products.

(For a continuation of this work see page 101.)

* Take 3 times 1 tenth of 345, or 1 tenth of 3 times 345.

† Lead the pupil to see that in problems of this kind, the correct result may be obtained by "*multiplying the numerators together for a new numerator and the denominators together for a new denominator*"; that in so doing he divides the multiplicand by the denominator of the multiplier and multiplies the quotient so obtained by the numerator of the multiplier.

Algebraic Fractions.

Review pp. 87 and 88.

$\frac{a}{b}+\frac{c}{d}$. l. c. d = bd	$\frac{2}{3}+\frac{5}{7}$. l. c. d = 21
$bd \div b = d$ $\quad \frac{a \times d}{b \times d} = \frac{ad}{bd}$	$21 \div 3 = 7$. $\frac{2 \times 7}{3 \times 7} = \frac{14}{21}$
$bd \div d = b$ $\quad \frac{c \times b}{d \times b} = \frac{bc}{bd}$	$21 \div 7 = 3$. $\frac{5 \times 3}{7 \times 3} = \frac{15}{21}$
$\frac{ad}{bd}+\frac{bc}{bd}=\frac{ad+bc}{bd}$	$\frac{14}{21}+\frac{15}{21}=\frac{29}{21}$

Observe that in cases like the above, in which the denominators are prime to each other, the l. c. d. is the product of the given denominators, and each new numerator may be found by multiplying the given numerator by the denominator of the other fraction.

191. Problems in addition and subtraction.

$$1.\ \frac{a}{x}+\frac{b}{y}=\frac{ay}{xy}+\frac{bx}{xy}=\frac{ay+bx}{xy}$$

Let $a = 2$, $b = 3$, $x = 5$, and $y = 7$, and verify.

$$2.\ \frac{a}{x}-\frac{b}{y}=\frac{ay}{xy}-\frac{bx}{xy}=\frac{ay-bx}{xy}.$$

Assign a numerical value to each letter and verify.

3. $\frac{2}{x}-\frac{1}{y}$.
4. $\frac{3}{x}-\frac{2}{y}$.

SOLVE.—Then let $x = 5$ and $y = 7$ and verify.

Algebraic Fractions.

192. Problems in Multiplication and Division.

Example I.

$$\frac{a}{b} \times c = \frac{ac}{b} \qquad \left| \qquad \frac{2}{5} \times 3 = \frac{6}{5} = 1\tfrac{1}{5} \right.$$

Let $a = 2$, $b = 5$, and $c = 3$; then $\frac{ac}{b} = \frac{2 \times 3}{5} = \frac{6}{5} = 1\frac{1}{5}$

Example II.

$$\frac{a}{b} \div c = \frac{a}{bc} \qquad \left| \qquad \frac{2}{5} \div 3 = \frac{2}{15} \right.$$

Let $a = 2$, $b = 5$, and $c = 3$; then $\frac{a}{bc} = \frac{2}{5 \times 3} = \frac{2}{15}$

Example III.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd} \qquad \left| \qquad \frac{2}{5} \times \frac{3}{7} = \frac{6}{35} \right.$$

Let $a = 2$, $b = 5$, $c = 3$, and $d = 7$; then $\frac{ac}{bd} = \frac{2 \times 3}{5 \times 7} = \frac{6}{35}$

I. Find the product and verify as above.

1. $\frac{a^2}{b^3} \times c$
2. $\frac{x}{y^3} \times x$
3. $\frac{x}{y^3} \times y$
4. $\frac{ab}{cd} \times a$
5. $\frac{ax}{bx} \times 3a$
6. $\frac{x}{7} \times 5$

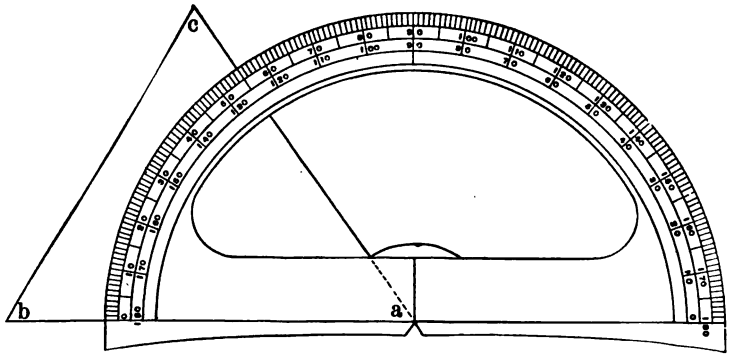
Observe that in the verification of these problems any value you please may be given to a letter only provided it retains the value given it, throughout the solution of that particular problem.

II. Find the quotient and verify as above.

1. $\frac{a^2}{b^3} \div c$
2. $\frac{x}{y^3} \div x$
3. $\frac{x}{y^3} \div y$
4. $\frac{ab}{cd} \div a$
5. $\frac{ax}{bx} \div 3a$
6. $\frac{x}{7} \div 5$

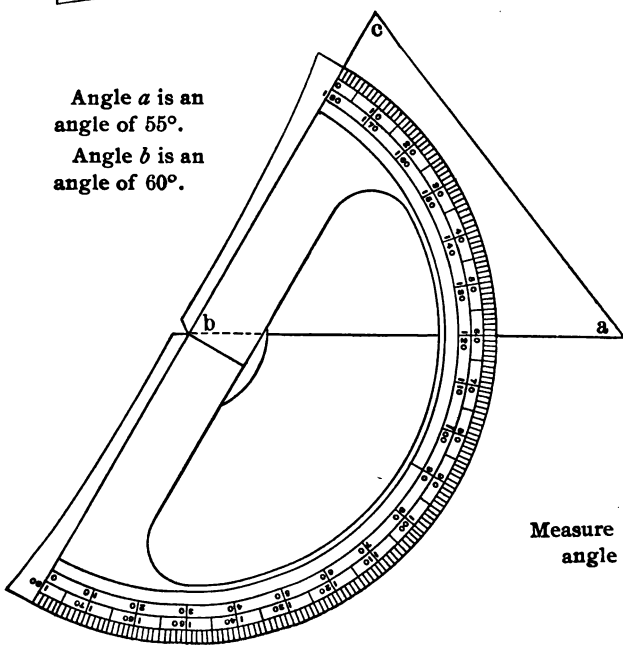
Geometry.

193. The Protractor.



Angle *a* is an angle of 55°.

Angle *b* is an angle of 60°.



Measure angle *c*.

194. Miscellaneous Review.

1. A piece of land in the form of an equilateral triangle measures on one side $46\frac{3}{11}$ rods. What is the distance around it?

2. The perimeter of a piece of land that is an exact square is $246\frac{3}{4}$ feet. How far across on one side?

3. The length of a certain rectangular field is three times its breadth; its perimeter is 360rd. What is its breadth? its length?

NOTE.—If its breadth is x feet then its length is $3x$ feet, and its perimeter is $2x + 6x$ feet. Then, $2x + 6x = 360$.

4. If $\frac{3}{7}$ of the value of a farm is \$2154, what is $\frac{7}{8}$ of the value of the farm?

NOTE.—If $\frac{3}{7}$ of a certain number is 24, what is the number? What is $\frac{7}{8}$ of the number?

5. I spent $\frac{3}{5}$ of my money and had \$3.60 remaining. (a) How much did I spend? (b) What I had remaining, equals what part of what I spent?

6. Change $\frac{5}{12}$ to an equivalent fraction whose denominator is 30.

7. Change $\frac{6}{7}$ to an equivalent fraction whose numerator is 30.

8. Change $\frac{a^2}{b}$ to an equivalent fraction whose denominator is bc.

9. Multiply $\frac{5}{9}$ by $\frac{3}{8}$ and multiply the product by 25.

10. Multiply $\frac{3}{40}$ by $\frac{1}{20}$ and multiply the product by 25.

11. If $\frac{3}{4}$ of an acre of land is worth \$36, how much are $37\frac{1}{2}$ acres worth at the same rate?

12. The rent of a house for 2 yr. 4 mo. was \$840. What was the rate per year?

FRACTIONS.

Continued from page 96.

195. To Divide by a Fraction.

Example I.

Divide 6 by $\frac{2}{3}$.

Operation No. 1.	Operation No. 2.
$6 = 1\frac{8}{3}$	$1 \div \frac{2}{3} = \frac{3}{2}$ †
$1\frac{8}{3} \div \frac{2}{3} = 9$ *	$6 \div \frac{2}{3} = 6$ times $\frac{3}{2} = 1\frac{8}{2} = 9$

Example II.

Divide $\frac{7}{8}$ by $\frac{3}{5}$.

Operation No. 1.	Operation No. 2.
$\frac{7}{8} = \frac{35}{40}$. $\frac{3}{5} = \frac{24}{40}$	$1 \div \frac{3}{5} = \frac{5}{3}$ §
$\frac{35}{40} \div \frac{24}{40} = 1\frac{11}{24}$ ‡	$\frac{7}{8} \div \frac{3}{5} = \frac{7}{8}$ of $\frac{5}{3} = \frac{35}{24} = 1\frac{11}{24}$

From the foregoing operations the following rules for dividing by a fraction are obtained:

Rule I.—*Reduce the dividend and the divisor to like fractional units, then divide the numerator of the dividend by the numerator of the divisor.*

Rule II.—"*Invert the divisor and proceed as in multiplication.*"

Observe that the inverted divisor shows the number of times the divisor is contained in 1: then in 6 it is contained 6 times as many times; in 4, 4 times as many; in $\frac{7}{8}$, $\frac{7}{8}$ as many; in $\frac{3}{4}$, $\frac{3}{4}$ as many, etc.

Note.—For another explanation of the inversion of the divisor, see Book II. page 232.

* 18 thirds ÷ 2 thirds = 9.

† 1 = $\frac{3}{3}$. 3 thirds ÷ 2 thirds = $1\frac{1}{2} = \frac{3}{2}$.

‡ 35 fortieths ÷ 24 fortieths = $1\frac{11}{24}$.

§ 1 = $\frac{5}{5}$. 5 fifths ÷ 3 fifths = $1\frac{2}{3} = \frac{5}{3}$.

Fractions.

I. Find the quotient. See page 106, problems 27 and 28.

1. $46 \div \frac{1}{2}$	4. $375 \div \frac{1}{2}$	7. $196 \div \frac{1}{2}$
2. $46 \div \frac{3}{4}$	5. $375 \div \frac{3}{4}$	8. $196 \div \frac{3}{4}$
3. $46 \div \frac{3}{5}$	6. $375 \div \frac{3}{5}$	9. $196 \div \frac{3}{5}$

(a) Find the sum of the nine quotients.*

II. Find the quotient. See page 106, problems 29 and 30.

1. $\frac{7}{8} \div \frac{2}{3}$	4. $\frac{5}{12} \div \frac{2}{3}$	7. $\frac{11}{24} \div \frac{2}{3}$
2. $\frac{7}{8} \div \frac{5}{6}$	5. $\frac{5}{12} \div \frac{5}{6}$	8. $\frac{11}{24} \div \frac{5}{6}$
3. $\frac{7}{8} \div \frac{10}{13}$	6. $\frac{5}{12} \div \frac{10}{13}$	9. $\frac{11}{24} \div \frac{10}{13}$

(b) Find the sum of the nine quotients.†

III. Find the quotient. See page 106, problems 31 and 32.

1. $5\frac{1}{2} \div \frac{3}{4}$	4. $24\frac{2}{3} \div \frac{3}{4}$	7. $19\frac{5}{6} \div \frac{3}{4}$
2. $5\frac{1}{2} \div \frac{6}{7}$	5. $24\frac{2}{3} \div \frac{6}{7}$	8. $19\frac{5}{6} \div \frac{6}{7}$
3. $5\frac{1}{2} \div \frac{2}{5}$	6. $24\frac{2}{3} \div \frac{2}{5}$	9. $19\frac{5}{6} \div \frac{2}{5}$

(c) Find the sum of the nine quotients.

IV. Find the quotient. See page 106, problem 33.

1. $325\frac{1}{2} \div 2\frac{1}{2}$	6. $174\frac{1}{2} \div 2\frac{1}{2}$
2. $325\frac{1}{2} \div 2\frac{2}{5}$	7. $174\frac{1}{2} \div 2\frac{2}{5}$
3. $325\frac{1}{2} \div 3\frac{1}{3}$	8. $174\frac{1}{2} \div 3\frac{1}{3}$
4. $325\frac{1}{2} \div 7\frac{1}{2}$	9. $174\frac{1}{2} \div 7\frac{1}{2}$
5. $325\frac{1}{2} \div 1\frac{1}{3}$	10. $174\frac{1}{2} \div 1\frac{1}{3}$

(d) Find the sum of the ten quotients.

* Why is the sum of the nine quotients equal to $(46 + 375 + 196) \times 5$?
† Why is the sum of the nine quotients equal to $(\frac{7}{8} + \frac{5}{12} + \frac{11}{24}) \times 4$?

Fractions.

196. To Reduce Complex Fractions to Simple Fractions.

Observe that every reduction of a complex fraction may be regarded as a problem in division.

$\frac{\frac{3}{4}}{\frac{7}{8}}$ is read, $\frac{3}{4}$ divided by $\frac{7}{8}$. $\frac{3}{4} \div \frac{7}{8} = \frac{3}{4}$ of $\frac{8}{7} = \frac{3}{4}\frac{8}{7} = \frac{6}{7}$.

I. Reduce to their simplest forms.

1. $\frac{\frac{5}{6}}{\frac{1}{3}}$ 2. $\frac{\frac{5}{6}}{\frac{3}{4}}$ 3. $\frac{\frac{5}{6}}{\frac{3}{5}}$ 4. $\frac{\frac{5}{6}}{\frac{1}{5}}$

(a) Find the sum of the four fractions.

Observe that a complex fraction may be reduced to a simple fraction by multiplying its numerator and denominator by some number that will in each case give an integral product. When this number can be easily discovered by inspection this is a convenient method of reduction: thus $\frac{\frac{1}{7}}{2} = \frac{\frac{1}{7} \times 2}{2 \times 2} = \frac{1}{14}$.

II. Reduce to their simplest forms. See page 106, problem 34.

1. $\frac{\frac{5}{6}}{5}$ 2. $\frac{\frac{1}{8}}{4}$ 3. $\frac{5}{\frac{1}{2}}$ 4. $\frac{1\frac{1}{2}}{2}$

(b) Find the sum of the four fractions.

III. Reduce to their simplest forms.

1. $\frac{8}{\frac{1}{2}}$ 2. $\frac{8}{\frac{3}{4}}$ 3. $\frac{8}{\frac{3}{5}}$ 4. $\frac{8}{\frac{1}{6}}$

(c) Find the sum of the four fractions.

IV. Reduce to their simplest form.

1. $\frac{12\frac{1}{2}}{100}$ * 2. $\frac{16\frac{2}{3}}{100}$ 3. $\frac{66\frac{2}{3}}{100}$ 4. $\frac{87\frac{1}{2}}{100}$

* Divide the numerator and denominator by $12\frac{1}{2}$.

Fractions.

197. Practical Application of the Preceding Rules.

Page 84, problems 1 and 2.

1. A man owned a farm of 375 acres; he gave to his eldest son 275 acres of his farm. What part of the farm did the son receive?

2. Peter Lundy earned $650 in one year; of this sum he expended $520. What part of his earnings did he expend?

Page 85, problems 1 and 2.

3. A lady owned $\frac{5}{8}$ of an acre of land. How many 160ths of an acre did she own?

4. Benton walked $\frac{1}{16}$ of a mile. Express the distance he walked in 160ths of a mile.

Page 91, problems 1 and 6.

5. In a certain furnace $\frac{5}{16}$ of a ton of coal was consumed in one day, and $\frac{7}{20}$ of a ton the next day. What part of a ton was consumed in the two days?

6. Mr. Luker has three lots of land; in the first lot there are $\frac{11}{16}$ of an acre; in the second, $\frac{1}{2}$ of an acre, and in the third, $\frac{13}{20}$ of an acre. How many acres in all?

Page 91, Art. 186, problems 3 and 4.

7. Of $\frac{9}{20}$ of a mile of board fence $\frac{1}{8}$ of a mile was burned. What part of a mile remained?

8. Mr. Reynolds had put into the bank $\frac{11}{20}$ of his annual salary; he drew from this money a sum equal to $\frac{1}{6}$ of his salary. What part of his salary remained in the bank?

Page 92, Art. 187, I., problems 4 and 5.

9. From $52\frac{5}{8}$ tons of hay, were sold and delivered $31\frac{3}{8}$ tons. How many tons remained of the unsold hay?

10. On Monday James rode on his wheel $47\frac{3}{8}$ miles; on Tuesday he rode $18\frac{3}{8}$ miles. How much further did he ride Monday than Tuesday?

Page 93, Art. 188, I., problems 1 and 5.

11. If a street car makes a round trip in $\frac{5}{12}$ of an hour, in how long a time can it make 4 such trips?

12. If $\frac{5}{4}$ yd. of ribbon are required to trim a hat, how much ribbon will be required to trim 8 such hats?

Page 93, Art. 188, II., problems 1 and 5.

13. At $3\frac{1}{2}$ dollars a cord, what is the cost of 7 cords of wood?

14. If Henry rides his wheel at the rate of $4\frac{2}{3}$ miles an hour, how far does he ride in 5 hours?

Page 94, Art. 189, I., problems 1 and 4.

15. If $\frac{7}{12}$ of a yd. of ribbon is cut into 4 equal pieces, what part of a yard is each piece?

16. John hoes 4 rows of corn in $\frac{5}{6}$ of an hour. In what part of an hour does he hoe 1 row?

Page 94, Art. 189, II., problems 1 and 5.

17. A horse traveled $17\frac{1}{3}$ miles in 3 hours. What was his rate per hour?

18. A farmer divided a field containing $18\frac{3}{10}$ acres into 4 equal lots. How many acres in each lot?

Page 96, Art. 190, I., problems 1 and 5.

19. At \$345 an acre, what is the cost of $\frac{1}{2}$ acre?

20. At \$263 an acre, what is the cost of $\frac{3}{4}$ acre?

Page 96, Art. 190, II., problems 1 and 9.

21. A piece of land in the form of a rectangle is $\frac{7}{16}$ of a mile long and $\frac{1}{2}$ of a mile wide. The piece is what part of a square mile?

22. At $\frac{1}{2}$ a dollar per yard, what is the cost of $\frac{3}{4}$ of a yard of silk?

Page 96, Art. 190, III., problems 1 and 9.

23. A strip of land $372\frac{1}{4}$ rods long and $\frac{1}{2}$ a rod wide contains how many square rods?

24. At $\$153\frac{1}{2}$ an acre, find the cost of $\frac{3}{4}$ of an acre.

Page 96, Art. 190, IV., problems 1 and 8.

25. At $\$462\frac{2}{3}$ a mile, what is the cost of grading $2\frac{1}{2}$ miles of road?

26. How many square feet in a piece of land $275\frac{1}{2}$ ft. by $4\frac{1}{2}$ ft.?

Page 102, I., problems 1 and 5.

27. At $\frac{1}{2}$ a dollar a bushel, how many bushels of potatoes can be bought for 46 dollars?

28. At $\frac{3}{4}$ of a dollar a bushel, how many bushels of apples can be bought for 375 dollars?

Page 102, II., problems 1 and 5.

29. At $\frac{2}{3}$ of a dollar a yard, how many yards of cloth can be bought for $\frac{7}{8}$ of a dollar?

30. At $\frac{5}{6}$ of a dollar a yard, what part of a yard of cloth can be bought for $\frac{5}{12}$ of a dollar?

Page 102, III., problems 1 and 6.

31. If a rectangular diagram on the blackboard contains $5\frac{1}{2}$ square feet and is $\frac{3}{4}$ of a foot wide, how long is it? Make the diagram.

32. At $\$\frac{2}{5}$ a bushel, how many bushels of meal can be bought for $\$24\frac{2}{3}$?

Page 102, IV., problem 1.

33. A strip of land contains $325\frac{1}{2}$ square rods, and is $2\frac{1}{2}$ rods wide. How long is it?

Page 103, II., problem 1.

34. Five-sixths of an hour is what part of 5 hours?

Algebraic Fractions.

Review pages 97 and 98.

198. PROBLEMS IN DIVISION WITH A FRACTION FOR A DIVISOR.

EXAMPLE I.

See page 101, Rule II, and Observation.

$$a \div \frac{b}{c} = \frac{ac}{b} \qquad \Big| \qquad 7 \div \frac{2}{3} = \frac{21}{2} = 10\tfrac{1}{2}$$

Let $a = 7$, $b = 2$, and $c = 3$; then $\frac{ac}{b} = \frac{7 \times 3}{2} = \frac{21}{2} = 10\frac{1}{2}$

EXAMPLE II.

See page 101, Rule II, and Observation.

$$\frac{a}{x} \div \frac{b}{c} = \frac{ac}{bx} \qquad \Big| \qquad \frac{7}{10} \div \frac{2}{3} = \frac{21}{20} = 1\tfrac{1}{20}$$

Let $a = 7$, $x = 10$, $b = 2$, and $c = 3$; then $\frac{ac}{bx} = \frac{7 \times 3}{2 \times 10} = \frac{21}{20} = 1\frac{1}{20}$

I. Find the quotient and verify as above.

1. $x \div \frac{a}{b}$ 3. $a \div \frac{b}{x}$ 5. $xy \div \frac{1}{a}$

2. $y \div \frac{c}{d}$ 4. $b \div \frac{c}{y}$ 6. $yz \div \frac{2}{a}$

II. Find the quotient and verify as above.

1. $\frac{a}{x} \div \frac{b}{y}$ 3. $\frac{c}{y} \div \frac{d}{z}$ 5. $\frac{b}{c} \div \frac{x}{y}$

2. $\frac{b}{x} \div \frac{c}{y}$ 4. $\frac{a}{z} \div \frac{b}{x}$ 6. $\frac{c}{d} \div \frac{x}{y}$

Algebraic Fractions.

199. Miscellaneous Exercises.

Example I.

$$a+\frac{b}{c}=\frac{ac+b}{c} \qquad \Big| \qquad 3\frac{2}{5}=\frac{17}{5}=3\frac{2}{5}$$

Let $a = 3$, $b = 2$, and $c = 5$.

Then $\frac{ac+b}{c}=\frac{(3\times 5)+2}{5}=\frac{17}{5}=3\frac{2}{5}$

I. Reduce to improper fractions and verify as above.

1. $x+\frac{a}{y}$ 3. $b+\frac{x}{2}$ 5. $3+\frac{2}{x}$
2. $y+\frac{2}{x}$ 4. $c+\frac{a}{x}$ 6. $x+\frac{2}{3}$

Example II.

$$\frac{ab+c}{b}=a+\frac{c}{b} \qquad \Big| \qquad \frac{11}{3}=3\frac{2}{3}$$

Let $a = 2$, $b = 3$, and $c = 5$; then $a+\frac{c}{b}=2+\frac{5}{3}=3\frac{2}{3}$

II. Reduce to mixed numbers and verify.

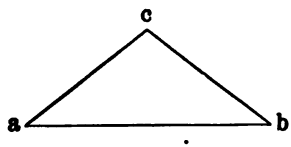
1. $\frac{ax+b}{x}$ 3. $\frac{by+2}{y}$ 5. $\frac{2ab+c}{2a}$
2. $\frac{by+c}{y}$ 4. $\frac{3y+a}{3}$ 6. $\frac{abc+d}{bc}$

III. Reduce and verify.

1. $\dfrac{\frac{a}{x}}{\frac{b}{y}}$ 2. $\dfrac{\frac{2}{a}}{\frac{3}{b}}$ 3. $\dfrac{\frac{x}{b}}{\frac{y}{c}}$ 4. $\dfrac{\frac{x}{3}}{\frac{y}{2}}$

Geometry.

200. Construction Problems. Triangles.



1. Draw a triangle. Make the side *ab* 3 inches long. Make the angle *a*, 45°. Make the angle *b*, 45°. Prove your work by measuring the angle *c* which should be an angle of —— degrees.

Observe that if two angles of any triangle and the length of the included side are given, the triangle may be drawn.*

2. Draw a triangle making one of the angles 40°, another 60°, and the included side 5 inches long. The third angle should measure —— degrees. Prove your work by measuring the third angle.

Measure the sides carefully and observe that the longest side is opposite the largest angle and the shortest side opposite the smallest angle.

3. Draw several triangles of different shapes and sizes. Convince yourself by measurement with the protractor that *the sum of the three angles of any triangle is —— degrees.*

4. Draw several triangles of different shapes and sizes. Convince yourself by measurement with a ruler that *the sum of two sides of any triangle is greater than the third side of the same triangle.*

5. Attempt to draw a triangle whose sides are 6 inches, $3\frac{1}{2}$ inches, and $2\frac{1}{2}$ inches.

*The "included side" is the side between the two given angles.

201. Miscellaneous Review.

1. If $\frac{2}{3}$ of a cord of wood cost $4.50 how much will 27 cords cost at the same rate?

2. If $2\frac{1}{2}$ tons of coal cost $12.60, how much will $17\frac{3}{5}$ tons cost at the same rate?

3. A man owned $7\frac{5}{10}$ acres of land; he sold $2\frac{3}{8}$ acres. What fractional part of his land did he sell?

4. The sum of two fractions is $1\frac{5}{12}$; one of the fractions is $\frac{1}{4}$. What is the other fraction?

5. The product of two fractions is $\frac{1}{6}\frac{5}{3}$; one of the fractions is $\frac{3}{7}$. What is the other fraction?

6. If a furnace consumes $\frac{3}{16}$ of a ton of coal a day, in how many days will $5\frac{1}{4}$ tons be consumed?

7. How many pounds of sugar at $4\cent$ a pound must be given for $27\frac{1}{2}$ pounds of butter at $23\cent$ a pound?

8. How many pounds of coffee at $33\frac{1}{3}\cent$ a pound must be given for $15\frac{1}{2}$ dozen eggs at $20\cent$ a dozen?

9. Which is the greater fraction $\frac{7}{8}$ or $\frac{7}{9}$?

10. Multiplying both terms of a fraction by the same number does not change the value of the fraction. Does adding the same number to both terms of a fraction change the value?

11. Dividing both terms of a fraction by the same number does not change the value of the fraction. Does subtracting the same number from both terms of a fraction change the value?

12. Change $\frac{5}{11\frac{1}{2}}$ to a fraction whose denominator is 46.

13. Change $\frac{3}{7\frac{1}{4}}$ to a fraction whose denominator is 15.

FRACTIONS.

202. To Change Decimals to Common Fractions and Common Fractions to Decimals.

Example I.

Change .36 to a common fraction in its lowest terms.*

$$.36 = \frac{36}{100} = \frac{9}{25}$$

Reduce to common fractions.

1. .45	3. .375	5. .55	7. .625
2. .045	4. .0375	6. .055	8. .0625

(a) Find the sum of the eight decimals.

(b) Find the sum of the eight common fractions.

Example II.

Change $\frac{3}{8}$ to a decimal.

Operation.	*Explanation.*
8)3.000	3 over 8 means 3 divided by 8. We therefore annex zeros to the numerator 3, and perform the division.
.375	

One eighth of 30 tenths is 3 tenths with a remainder of 6 tenths, etc.

Reduce to decimals.

1. $\frac{2}{4}$	3. $\frac{2}{5}$	5. $\frac{1}{4}$	7. $\frac{2}{5}$
2. $\frac{3}{40}$	4. $\frac{3}{50}$	6. $\frac{21}{40}$	8. $\frac{41}{50}$

(c) Find the sum of the eight decimals.

*The numerator of .36 is 36; its denominator is 100, and the decimal is 36 hundredths.

Fractions.

EXAMPLE III.

Change $\frac{2}{7}$ to a decimal.

Operation.

$$7)\underline{2.00}$$
$$.28 +$$

$$7)\underline{2.000}$$
$$.285 +$$

$$7)\underline{2.0000}$$
$$.2857 +$$

Explanation.

It will be observed that, however far this division may be carried, there is always a remainder. The fact that there is a remainder is indicated by writing the plus sign after the last figure of the decimal. The first quotient may be read *28 hundredths, plus.*

Observe, too, that the error in the first answer is less than 1 hundredth, since the true quotient is more than 28 hundredths and less than 29 hundredths. We may therefore say that the first result is *true to hundredths;* the second, *true to thousandths*, etc.

Reduce to decimals, true to thousandths.

1. $\frac{3}{7}$	4. $\frac{2}{11}$	7. $\frac{1}{6}$	10. $\frac{5}{8}$
2. $\frac{5}{7}$	5. $\frac{3}{11}$	8. $\frac{5}{6}$	11. $\frac{2}{3}$
3. $\frac{6}{7}$	6. $\frac{6}{11}$	9. $\frac{4}{9}$	12. $\frac{1}{3}$

(a) Find the sum of the twelve common fractions.

(b) Find the sum of the twelve decimals.*

Determine which of the following fractions can be reduced to terminating decimals † and which cannot.

1. $\frac{2}{3}$	4. $\frac{5}{6}$	7. $\frac{2}{5}$	10. $\frac{5}{12}$	13. $\frac{2}{15}$
2. $\frac{3}{4}$	5. $\frac{2}{7}$	8. $\frac{3}{10}$	11. $\frac{3}{13}$	14. $\frac{3}{16}$
3. $\frac{2}{5}$	6. $\frac{3}{8}$	9. $\frac{2}{11}$	12. $\frac{5}{14}$	15. $\frac{7}{20}$

Observe that if a fraction is in its lowest terms and the denominator contains any other prime factor besides 2's and 5's, the fraction cannot be reduced to a "terminating" decimal. Can you tell why?

* Observe that the difference between *a* and *b* must be less than 12 thousandths. Why?

† See footnote, page 113.

Fractions.

203. To Reduce a Complex Decimal* to a Common Fraction.

Example.

Change $.27\frac{3}{11}$ to a common fraction.

Operation.	*Explanation.*
$.27\frac{3}{11} = \frac{27\frac{3}{11}}{100}$	Writing the denominator, we have $27\frac{3}{11}$ divided by 100.
$27\frac{3}{11} = \frac{300}{11}$	$27\frac{3}{11}$ reduced to 11ths $= \frac{300}{11}$
$\frac{300}{11} \div 100 = \frac{300}{1100} = \frac{3}{11}$	$\frac{300}{11} \div 100 = \frac{300}{1100} = \frac{3}{11}$

Reduce to common fractions.

1. $.38\frac{1}{3}$	4. $.24\frac{2}{11}$	7. $.61\frac{2}{3}$
2. $.83\frac{1}{6}$	5. $.35\frac{3}{11}$	8. $.16\frac{5}{6}$
3. $.45\frac{5}{7}$	6. $.40\frac{6}{11}$	9. $.54\frac{5}{7}$

(a) Find the sum of the nine common fractions.

(b) Find the sum of the nine complex decimals.

Note.—If the division be carried sufficiently far in any non-terminating decimal there will be found a certain figure or set of figures that is constantly repeated: thus, we may have, .3666666, or .27272727 or .5236236236. The part repeated is called *a repetend*, and may be written thus: $.3\dot{6}$, $.\dot{2}\dot{7}$, $.5\dot{2}3\dot{6}$. It is a curious fact that the real denominator of any repetend is as many 9's as there are figures in the repetend; $.3\dot{6} = .3\frac{6}{9}$, $.\dot{2}\dot{7} = \frac{27}{99}$, $.5\dot{2}3\dot{6} = .5\frac{236}{9999}$.

* Decimals that are complete without the annexation of a common fraction are said to be *terminating decimals*. .24 is a terminating decimal. .66.666 + is a non-terminating decimal. A decimal with a common fraction annexed, as $.3\frac{1}{3}$, is sometimes called a *complex decimal*.

Fractions.

204. To Reduce a Fraction to Hundredths.

Examples.

I. $\frac{3}{4}$ $\quad 4)3.00$ $\quad .75$; or $\frac{3}{4}$ of $\frac{100}{100} = \frac{75}{100}$ or .75

II. $\frac{2}{3}$ $\quad 3)2.00$ $\quad .66\frac{2}{3}$; or $\frac{2}{3}$ of $\frac{100}{100} = \frac{66\frac{2}{3}}{100}$ or $.66\frac{2}{3}$

III. $\frac{7}{8}$ $\quad 8)7.00$ $\quad .87\frac{1}{2}$; or $\frac{7}{8}$ of $\frac{100}{100} = \frac{87\frac{1}{2}}{100}$ or $.87\frac{1}{2}$

IV. $\frac{1}{40}$ $\quad 40)1.00$ $\quad .02\frac{1}{2}$; or $\frac{1}{40}$ of $\frac{100}{100} = \frac{2\frac{1}{2}}{100}$ or $.02\frac{1}{2}$

Observe that this work is not different from that given under example II., page 111., except that the number of zeros to be annexed to the numerator *is always two.* Why?

I. Reduce to hundredths.

1. $\frac{1}{2}$	6. $\frac{1}{4}$	11. $\frac{1}{10}$	16. $\frac{3}{20}$
2. $\frac{1}{3}$	7. $\frac{1}{6}$	12. $\frac{2}{5}$	17. $\frac{5}{6}$
3. $\frac{1}{8}$	8. $\frac{3}{8}$	13. $\frac{11}{40}$	18. $\frac{9}{40}$
4. $\frac{3}{10}$	9. $\frac{2}{5}$	14. $\frac{7}{25}$	19. $\frac{1}{50}$
5. $\frac{1}{7}$	10. $\frac{2}{7}$	15. $\frac{3}{14}$	20. $\frac{5}{14}$

(a) Find the sum of the twenty decimals.

(b) Find the sum of the twenty common fractions.

II. Reduce to hundredths.

1. $\frac{11}{175}$	4. $\frac{18}{175}$	7. $\frac{24}{175}$
2. $\frac{15}{175}$	5. $\frac{21}{175}$	8. $\frac{16}{175}$
3. $\frac{17}{175}$	6. $\frac{35}{175}$	9. $\frac{18}{175}$

(a) Find the sum of the nine decimals.

(b) Find the sum of the nine common fractions.

Fractions.

III. Reduce to hundredths.

NOTE.—Such fractions as the following may be easily reduced to hundredths by dividing the numerator and the denominator of each by that number which will change the denominator to 100.*

1. $\frac{37}{500}$
2. $\frac{46}{500}$
3. $\frac{94}{500}$
4. $\frac{323}{500}$
5. $\frac{45}{400}$
6. $\frac{92}{400}$
7. $\frac{120}{400}$
8. $\frac{143}{400}$
9. $\frac{36}{300}$
10. $\frac{40}{300}$
11. $\frac{85}{300}$
12. $\frac{139}{300}$

(a) Find the sum of the twelve results.

(b) Find the sum of the twelve common fractions.

IV. Reduce to hundredths.

NOTE.—Multiply the numerator and the denominator of each fraction by that number which will change the denominator to 100*.

1. $\frac{7}{20}$
2. $\frac{14}{25}$
3. $\frac{27}{50}$
4. $\frac{8}{12\frac{1}{2}}$
5. $\frac{7}{10}$
6. $\frac{4\frac{1}{2}}{12\frac{1}{2}}$
7. $\frac{13}{20}$
8. $\frac{11}{25}$
9. $\frac{23}{50}$

(a) Find the sum of the nine decimals.

V. Reduce to hundredths.

1. $\frac{115}{250}$†
2. $\frac{69}{150}$
3. $\frac{154}{350}$
4. $\frac{6}{40}$†
5. $\frac{18}{40}$
6. $\frac{16}{40}$
7. $\frac{135}{250}$
8. $\frac{81}{150}$
9. $\frac{126}{350}$

(a) Find the sum of the nine decimals.

* Every common fraction can be changed to hundredths by annexing two zeros to the numerator and dividing by the denominator; but this method of reduction is not always the most simple.

† Take $\frac{1}{2}$ of the numerator and $\frac{1}{2}$ of the denominator.

‡ Multiply the numerator and the denominator by $2\frac{1}{2}$.

Fractions.

205. Denominate Fractions.

1. One half inch is what part of a foot?
2. Two and $\frac{1}{2}$ inches are what part of a foot?
3. Five and $\frac{1}{2}$ inches are what part of a foot?

4. One half foot is what part of a rod?
5. Three and one half feet are what part of a rod?
6. Ten and one half feet are what part of a rod?

7. Sixty-four rods are what part of a mile?
8. Ninety-six rods are what part of a mile?
9. One hundred eighty rods are what part of a mile?

10. One and one half quarts are what part of a peck?
11. Two and one half quarts are what part of a gallon?
12. Twenty-four quarts are what part of a bushel?

13. Fourteen ounces are what part of a pound?
14. Seven and one half ounces are what part of a pound?
15. One and one fourth ounces are what part of a pound?

16. Six hundred pounds are what part of a ton?
17. Four hundred fifty pounds are what part of a ton?
18. Six hundred twenty-five pounds are what part of a ton?

19. Seventy-five square rods are what part of an acre?
20. Forty-five square rods are what part of an acre?
21. One hundred square rods are what part of an acre?

22. Thirty-two cubic feet are what part of a cord?
23. Fifty-six cubic feet are what part of a cord?
24. One hundred cubic feet are what part of a cord?

25. Seven and one half minutes are what part of an hour?
26. Forty minutes are what part of an 8-hour day?
27. Ninety minutes are what part of an 8-hour day?

Algebraic Fractions.

(Review pages 77 and 78).

206. Fractions in Equations.

Example I.

$$\frac{x}{2} + 2x = 30$$

Multiplying both members of the equation (See page 77, Art. 170, Statement 5) by 2, the denominator of the fraction in the equation, we have—

$$x + 4x = 60$$

Uniting terms $\quad 5x = 60$

Dividing by 5 $\quad x = 12$

Example II.

$$\frac{x}{2} + \frac{x}{3} + 5x = 70$$

Multiplying by 2 $\quad x + \frac{2x}{3} + 10x = 140$

Multiplying by 3 $\quad 3x + 2x + 30x = 420$*

Uniting terms $\quad 35x = 420$

Dividing by 35 $\quad x = 12$

Problems.

Find the value of x.

1. $\frac{x}{5} + \frac{x}{3} = 8$

2. $\frac{x}{2} - \frac{x}{3} = 6$

3. $\frac{x}{3} + 5x = 64$

4. $2x - \frac{x}{3} = 50$

*Observe that the equation might have been cleared of fractions by multiplying both its members by 6, the l. c. m. of 2 and 3.

Algebra.

207. Problems Leading to Equations Containing One Unknown Quantity Without Fractions.

Example.

John and Henry together have 60 oranges, and Henry has three times as many as John. How many has each?

Let x = the number John has,
then $3x$ = the number Henry has,
and $x + 3x$ = the number they together have.

But they together have 60.

Therefore $x + 3x = 60$.
Uniting $4x = 60$.
Dividing $x = 15$.
Multiplying $3x = 45$.

Therefore John has 15 oranges and Henry has 45 oranges.

Problems.

1. The sum of two numbers is 275, and the greater is four times the less. What are the numbers?

2. Robert has a certain sum of money and Harry has five times as much; together they have $216. How many dollars has each?

3. One number is four times another, and their difference is 270. What are the numbers?

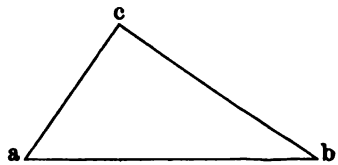
4. Peter has a certain number of marbles and William has 8 more than Peter; together they have 96 marbles. How many has each?

5. Sarah has a certain number of pennies and her sister has nine more than twice as many; together they have 93. How many has each?

6. Two times Reuben's money plus three times his money equals 175 dollars. How many dollars has he?

Geometry.

208. Construction Problems—Triangles.



1. Draw a triangle whose base, *ab*, is 3 inches long. Make the angle *a*, 55° and the angle *b*, 35°. The angle *c* should be — — degrees. Measure the three sides.

Observe that the longest side is opposite the greatest angle and the shortest side opposite the smallest angle.

2. Draw a triangle two of whose sides are equal. Measure and compare the angles opposite the equal sides.

Observe that a triangle, two of whose sides are equal, has two angles equal; and conversely if two angles of a triangle are equal, two of the sides are equal.

3. If two triangles have the three sides of one equal to the three sides of the other, each to each, do you think the two triangles are alike in every respect?

4. If two triangles have the three angles of one equal to the three angles of the other, each to each, do you think the two triangles are necessarily alike in every respect?

5. Draw two triangles, the angles of one being equal to the angles of the other, and the sides of one *not* being equal to the sides of the other.

6. Is it possible to draw a triangle whose sides are equal, but whose angles are unequal?

209. Miscellaneous Review.

1. Without a pencil, change each of the following fractions to hundredths: $\frac{3}{5}$, $\frac{1}{8}$, $\frac{5}{8}$, $\frac{7}{10}$, $\frac{3}{25}$, $\frac{7}{50}$, $\frac{5}{12\frac{1}{2}}$, $\frac{7}{33\frac{1}{3}}$, $\frac{9}{16\frac{2}{3}}$, $\frac{60}{500}$, $\frac{30}{200}$, $\frac{45}{900}$, $\frac{60}{150}$,* $\frac{60}{250}$, $\frac{60}{125}$, $\frac{60}{66\frac{2}{3}}$.†

2. Butter that cost 25¢ a pound was sold for 29¢ a pound. The gain was equal to what part of the cost? The gain was equal to how many hundredths of the cost?

3. The taxes on an acre of land which was valued at $600, were $12. The taxes were equal to what part of the valuation? The taxes were equal to how many hundredths of the valuation?

4. Mr. Jones purchased 500 barrels of apples. He lost by decay a quantity equal to 75 barrels. What part of his apples did he lose? How many hundredths of his apples did he lose?

5. Regarding a month as 30 days and a year as 360 days, what part of a year is 7 months and 10 days? How many hundredths of a year in 7 months and 10 days? How many thousandths of a year? How many ten-thousandths of a year?

6. One cord 48 cubic feet is what part of 4 cords 16 cubic feet? Change the fraction to hundredths; to thousandths; to ten-thousandths.

7. One mile 240 rods is what part of 3 miles 160 rods? Change the fraction to hundredths; to thousandths; to ten-thousandths.

8. From a bill of $175 there was a discount of $14. The discount is equal to how many hundredths of the amount of the bill?

* Since 100 is $\frac{5}{3}$ of 150, take $\frac{5}{3}$ of 60. † 100 is $1\frac{1}{2}$ times $66\frac{2}{3}$.

PERCENTAGE.

210. Per cent means *hundredth* or *hundredths.* Per cent may be expressed as a common fraction whose denominator is 100, or it may be expressed decimally; thus, 6 per cent = $\frac{6}{100}$ or .06; $28\frac{1}{2}$ per cent = $\frac{28\frac{1}{2}}{100}$ or $.28\frac{1}{2}$; $\frac{1}{2}$ per cent = $\frac{\frac{1}{2}}{100}$ or $.00\frac{1}{2}$.

NOTE.—Instead of the words *per cent* sometimes the sign (%) is used; thus 6 per cent may be written 6 %.

211. The **base** in percentage is the number of which hundredths are taken; thus, in the problem, *Find 11 % of 600*, the base is 600; in the problem, *16 is what per cent of 800?* the base is 800; in the problem, *18 is 3 % of what?* the base is not given, but is to be found by the student.

Observe that whenever the base is given in problems like the above, it follows the word *of.*

212. There are three cases in percentage and only three.

Case I. To find some per cent (hundredths) of a number, as: find 15 % of 600.

Case II. To find a number when some per cent of it is given, as: 24 is 8 % of what number?

Case III. To find what per cent one number is of another, as: 12 is what % of 400?

Observe that a thorough knowledge of fractions is the necessary preparation for percentage. The work in percentage is work in fractions, *the denominator employed being 100.*

Percentage.

213. CASE I.

Find 17 per cent (.17) of 8460.

NOTE 1.—No new rule is necessary for performing this operation. We may find $\frac{3}{4}$ of a number by finding 3 times 1 fourth of it; that is, by multiplying it by $\frac{3}{4}$. So we may find .17 of a number by finding 17 times 1 hundredth of the number; that is, by multiplying by .17.

Operation.	*Explanation.*
84 60 (v) .17 592.20 846.0 1438.20	One per cent (1 hundredth) of 8460 is 84.60; 17 per cent (hundredths) of 8460 is 17 times 84.60, or 1438.20.

PROBLEMS.

1. Find 17% of 6420; of 5252; of 31.40.
2. Find 35% of 6420; of 5252; of 31.40.
3. Find 43% of 6420; of 5252; of 31.40.
4. Find 25% of 6420; of 5252; of 31.40.
5. Find 50% of 6420; of 5252; of 31.40.
6. Find 14% of 6420; of 5252; of 31.40.
7. Find 16% of 6420; of 5252; of 31.40.
8. Find 35% of 6420; of 5252; of 31.40.
9. Find 65% of 6420; of 5252; of 31.40.

(a) Find the sum of the twenty-seven results.

10. A sold goods for B. As remuneration for his services he received a sum equal to 12% of the sales. He sold $2146 worth of goods. How much did he receive for his services?

11. C is a collector of money. For this service he charges a commission of 6%; that is, his pay is 6% of the amount collected. He collected for D $375. How much should he pay over to D, and how much should he retain as pay for collecting?

Percentage.

214. CASE II.

673.20 is 17 per cent (.17) of what number?

Operation No. 1.

17)673.20(39.60
51 100
163 3960.
153
102
102

Explanation.

Since 673.20 is 17 hundredths of the number, 1 hundredth of the number is 1 seventeenth of 673.20, or 39 60; and 100 hundredths = 100 times 39.60, or 3960.

Operation No. 2.

.17)673.20(3960
51
163
153
102
102

Explanation.

We may find 100 seventeenths of a number by finding 1 seventeenth of a hundred times the number. 100 times 673.20 is 67320. 1 seventeenth of 67320 is 3960.

Observe that in the second operation we simply divide the number by 17 hundredths. Dividing by .17 is finding $\frac{100}{17}$ of the dividend, just as dividing by $\frac{3}{4}$ is finding $\frac{1}{3}$ of the dividend, and dividing by $\frac{1}{4}$ is finding $\frac{2}{1}$ of the dividend.

NOTE.—Sometimes the process may be shortened by writing the per cent as a common fraction and reducing it to its lowest terms; then using the reduced fraction instead of the one whose denominator is 100.

PROBLEMS.

1. 360 is 15 % of what number?
2. 360 is 25 % of what number?
3. 360 is 50 % of what number?
4. 360 is 75 % of what number?
5. 360 is 40 % of what number?
6. 360 is 3 % of what number?
7. 360 is 2 % of what number?

(a) Find the sum of the seven results.

Percentage.

CASE II.—*Continued.*

PROBLEMS.

1. $34.32 is 13 per cent of what?
2. $34.32 is 15 per cent of what?
3. $34.32 is 25 per cent of what?
4. $34.32 is 33⅓ per cent of what?
5. $34.32 is 50 per cent of what?

6. $64.98 is 19 per cent of what?
7. $64.98 is 12 per cent of what?
8. $64.98 is 20 per cent of what?
9. $64.98 is 24 per cent of what?
10. $64.98 is 25 per cent of what?

(a) Find the sum of the ten results.

11. A sold goods for B. As remuneration for his services A received a sum equal to 12% of the sales. He received $33.06. What was the amount of his sales? How much money does B have left of what he received for the goods after paying out of it A's commission?

12. C is a collector of money. For this service he charges a commission of 6%; that is, his pay is 6% of the amount collected. His commission on a certain collection was $74.40. What was the amount collected? How much should the man for whom he collected the money, receive?

13. C collected a sum of money for D, deducted his commission of 6%, and paid the remainder of the sum collected to D. He paid D $350.15*. What was the sum collected? How much money did C retain as his commission for collecting?

* $350.15 is what % of the amount collected?

Percentage.

215. Case III.

625 is what per cent of 900?

Operation No. 1.

$$\begin{array}{r} 900)625'00(.69\frac{4}{9} \\ 5400 \\ \hline 8500 \\ 8100 \\ \hline 400 = \frac{4}{9} \end{array}$$

Explanation.

625 is $\frac{625}{900}$ of 900.

This fraction may be changed to hundredths in the usual manner, viz., by performing the division indicated, "*carrying out*" *to hundredths only.*

Operation No. 2.

$\frac{625}{900} = \frac{25}{36}$

$$\begin{array}{r} 36)25'00(.69\frac{4}{9} = 69\frac{4}{9}\% \\ 216 \\ \hline 340 \\ 324 \\ \hline 16 = \frac{4}{9} \end{array}$$

Explanation.

625 is $\frac{625}{900}$ or $\frac{25}{36}$ of 900.

The fraction, $\frac{25}{36}$, may be changed to hundredths in the usual manner.

Operation and Explanation No. 3.

625 is $\frac{625}{900}$ of 900. $\frac{625 \div 9}{900 \div 9} = \frac{69\frac{4}{9}}{100} = .69\frac{4}{9} = 69\frac{4}{9}\%$

Operation and Explanation No. 4.

One hundredth of 900 is 9; then 625 is as many hundredths of 900, as 9 is contained times in 625. 9 is contained in 625, $69\frac{4}{9}$ times; so 625 is $69\frac{4}{9}$ % (hundredths) of 900.

Solve each of the following problems by each of the methods given above.

What per cent (how many hundredths) of 800 is--

(1) 250? (2) 375? (3) 475? (4) 350? (5) 150?

(a) Find the sum of the five results.

Percentage.

CASE III.—*Continued.*

PROBLEMS.

What % is—

1. 32 of 600?	6. 50 of 750?	11. 40 of 325?
2. 54 of 600?	7. 90 of 750?	12. 50 of 325?
3. 75 of 600?	8. 85 of 750?	13. 62 of 325?
4. 95 of 600?	9. 80 of 750?	14. 78 of 325?
5. 344 of 600?	10. 445 of 750?	15. 95 of 325?

(a) Find the sum of the first five results.

(b) Find the sum of the second five results.

(c) Find the sum of the third five results.

What % is—

16. \$.76 of \$38? (What is one % of \$38?)
17. \$9.02 of \$225.50? (What is one % of \$225.50?)
18. \$17.13 of \$342.60? (What is one % of \$342.60?)
19. \$58.45 of \$835? (What is one % of \$835?)
20. \$1.29 of \$6.45? (What is one % of \$6.45?)
21. \$113.10 of \$754? (What is one % of \$754?)
22. \$21 of \$175? (What is one % of \$175?)

23. A sold goods for B to the amount of \$346.25; he received as his commission for selling the goods, \$41.55. His commission was what per cent of the sales?

24. C collected for D \$643.50; he retained as his commission for collecting the money, \$38.61. His commission was what per cent of the sales?

25. F purchased 256 barrels of apples; he lost by decay a quantity equal to 16 barrels. What % of his apples did he lose?

Algebra.

(Review Page 117.)

216. Problems Leading to Equations Containing Fractions.

If to $\frac{1}{3}$ of Bernie's money you add $\frac{1}{4}$ of his money the sum is $63. How much money has he?

Let x = the number of dollars he has.

Then $\frac{x}{3} + \frac{x}{4} = 63$

Multiplying by 3, $x + \frac{3x}{4} = 189$

Multiplying by 4, $4x + 3x = 756$ *

Uniting terms, $7x = 756$

Dividing by 7, $x = 108$

Thus we find that Bernie has 108 dollars.

1. If to $\frac{1}{2}$ of a certain number you add $\frac{1}{5}$ of the same number, the sum is 45. What is the number?

2. A lady upon being asked her age replied: If from $\frac{1}{2}$ of my age you subtract $\frac{1}{5}$ of my age the remainder will be 9 years. How old was she?

3. The sum of two numbers is 72, and the less number equals $\frac{3}{5}$ of the greater number. What are the numbers?

Let x = the greater number.

4. The sum of two numbers is 75, and the less number equals $\frac{2}{3}$ of the greater number. What are the numbers?

5. The difference of two numbers is 30, and the less number equals $\frac{3}{5}$ of the greater number. What are the numbers?

* Observe that the equation might have been cleared of fractions by multiplying both its members by 12, the l. c. m. of 3 and 4.

Algebra.

217. Miscellaneous Problems.

1. Harry has some marbles; Joseph has 8 more than $\frac{1}{2}$ as many as Harry; together they have 68. How many has each?

2. William has 12 more than $\frac{1}{2}$ as many cents as Lucius; together they have 87 cents. How many cents has each?

3. If to the half of a certain number you add a third of the number, the sum will be 5 more than 3 fourths of the number. What is the number?

Let x = the number.

Then $\frac{x}{2} + \frac{x}{3} - 5 = \frac{3x}{4}$

4. If to $\frac{1}{2}$ of a number you add $\frac{1}{4}$ of the same number, the sum will be 12 more than $\frac{3}{8}$ of the number. What is the number?

5. If to a certain number you add 5 times the number, the sum will be 24 more than 4 times the number. What is the number?

6. If to $\frac{1}{2}$ of a number you add $\frac{1}{3}$ of the number the sum will be a. What is the number?

Let x = the number.

Then $\frac{x}{2} + \frac{x}{3} = a$

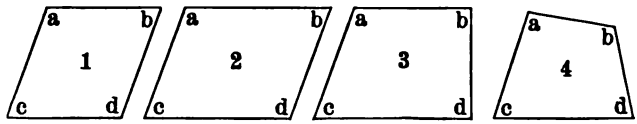
Multiplying by 6,	$3x + 2x = 6a$
Uniting	$5x = 6a$
Dividing by 5	$x = \frac{6a}{5}$

Observe that you may put in the place of a any number you please; hence any number is $\frac{6}{5}$ of the sum of its half and third.

Reason in a similar manner with reference to the half and fourth of a number; to the third and fourth.

Geometry.

218. Construction Problems— Quadrilaterals.



1. Draw a rhombus. Make the angle *a*, 110°. Use the protractor for measuring the angle *a* only. Make *ab* and *ac* each 2 inches. Then draw *bd* parallel to *ac* and *cd* parallel to *ab*. Prove your work by measuring the other angles.

Observe that when one side and one angle of a rhombus are given, the rhombus may be drawn.

2. If the angle *a* of a rhomboid is 110°, how many degrees in angle *b*? in angle *c*? in angle *d*?

3. Draw a trapezoid. Make the side *ab* 3 inches long. Make the angle *a*, 110 degrees, and the line *ac* 2 inches long. Make the angle *b*, 90 degrees; and the line *bd* of indefinite length, but long enough to form the side *bd*. Draw *cd* parallel to *ab*. Prove the work by finding the sum of the four angles.

4. Draw a trapezium. Make the angle *a*, 100°; the line *ab* 3 inches long, and the line *ac* 2 inches long. Make the angle *b*, 120°, and the side *bd* of indefinite length. Make the angle *c*, 85°. Angle *d* should be an angle of how many degrees? Prove the work by measuring angle *d*.

5. Draw four lines that together make a very irregular trapezium; then using the protractor, find the sum of the four angles.

219. Miscellaneous Review.

1. If I lose $\frac{3}{5}$ of my money, what % of my money do I lose?

2. Mr. Button spent $5 of the $12 which he had earned. What % of what he earned did he spend?

3. Mr. Thomas earns $150 per month. The monthly rent of the house in which he lives is equal to 15 % of what he earns. How much is his rent per year?

4. Mr. Jones purchased 500 barrels of apples. He lost by decay a quantity equal to 75 barrels. What per cent of the apples purchased remained sound? *

5. Mr. Brown borrowed $625. He used 87 % of this money to pay debts. How much of the money borrowed did he have left?

6. Mr. Green paid $24.60 for a suit of clothes. If this was exactly 15 % of his monthly earnings, how much does he earn per month?

7. Mr. White earns $1500 per year. He spends $312 for board, $200 for clothes, $75 for books and papers, $80 for traveling and amusements; gives to his church $40, for charitable purposes, $35, and all his other expenses amount to $23. The remainder of his salary he puts into a savings bank. What per cent of his salary does he save?

8. If to $\frac{1}{2}$ of my money you add $\frac{1}{5}$ of my money the sum will be $63. How much money have I?

9. The sum of two numbers is 120, and the less number is $\frac{3}{5}$ of the greater. What are the numbers?

10. If one angle of a rhomboid is 80°, what is the size of each of the other angles?

11. Three twenty-fifths is what % of $\frac{3}{5}$?

* See problem 4, page 120.

PERCENTAGE.

(Continued from page 126.)

220. Problems in Case III. Somewhat Disguised.

Example No. 1.

75 is how many % more than 60?

75 is 15 more than 60; so the percentage part of this problem is, *15 is what % of 60?* Ans. 25 %. Therefore 75 is 25 % more than 60; that is, 75 is 25 hundredths of 60 more than 60.

Example No. 2.

60 is how many % less than 75?

60 is 15 less than 75; so the percentage part of this problem is, *15 is what % of 75?* Ans. 20 %. Therefore 60 is 20 % less than 75; that is, 60 is 20 hundredths of 75 less than 75.

Observe that in example No. 1, 60 is the base, and that in example No. 2, 75 is the base; and that in problems of this kind the base always follows the word *than*.

Problems.

1. 60 is how many per cent more than 45?
2. 45 is how many per cent less than 60?
3. 150 is how many per cent more than 125?
4. 125 is how many per cent less than 150?
5. 225 is how many per cent more than 200?
6. 200 is how many per cent less than 225?
7. James has $345; Peter has $414. Peter has how many per cent more than James?* James has how many per cent less than Peter?†

* James's money is the base.

† Peter's money is the base.

Percentage.

221. Application of Art. 220 to "Loss and Gain."

Note.—In speaking of the per cent of loss or of gain the *cost* is regarded as the *base* unless otherwise specified.

Find the per cent of loss or gain in each of the following:

1. Bought for 25¢ and sold for 30¢.
2. Bought for 25¢ and sold for 23¢.
3. Bought for 25¢ and sold for 27¢.
4. Bought for 25¢ and sold for 21¢.
5. Bought for $40 and sold for $48.
6. Bought for $40 and sold for $35.
7. Bought for $40 and sold for $52.
8. Bought for $40 and sold for $36.
9. Sold for 65¢ that which cost 50¢.
10. Sold for 18¢ that which cost 20¢.
11. Sold for 30¢ that which cost 36¢.
12. Sold for 35¢ that which cost 30¢.
13. Sold for $50 that which cost $40.
14. Sold for $40 that which cost $50.
15. Sold for $30 that which cost $40.
16. Sold for $40 that which cost $30.

17. Mr. Watson bought 60 lbs. of tea at 32¢ a pound and sold it at 50¢. What was his per cent of gain if he sold as many pounds as he bought? If he lost 4 lb. by "down-weights" and wastage, how much money did he gain? What was his real per cent of gain?

18. Mr. Jenkins bought gloves at $4.50 per dozen and sold them at 50¢ a pair. What was his per cent of gain?

19. Mr. Warner bought apples at 40¢ a bushel. He lost 25 % of them by decay and sold the remainder at 50¢ a bushel. Did he gain or lose by the transaction? What was his per cent of gain or loss?

Percentage.

222. PERCENTAGE PROBLEMS UNDER CASE I, IN WHICH THE PER CENT IS MORE THAN 100.

Find 175% ($1\frac{75}{100}$ or 1.75) of \$632.60.

Operation No. 1.

\$632.60	
1.75	
\$31.6300	
\$442.820	
\$632.60	
\$1107.0500	

Explanation.

One % of \$632.60 is \$6.326.

175% of \$632.60 is 175 times \$6.326 or \$1107.05.

Observe that 5% of \$632.60 is \$31.63; that 70% of \$632.60 is \$442.82; that 100% of \$632.60 is \$632.60.

Operation No. 2.

$175\% = 1\frac{75}{100} = \frac{7}{4} \cdot \frac{1}{4}$ of \$632.60 = \$158.15.

$\frac{7}{4}$ of \$632.60 = 7 times \$158.15 or \$1107.05.

PROBLEMS.

1. Find 175% of 356; of 276; of 540.20
2. Find 155% of 356; of 276; of 540.20
3. Find 145% of 356; of 276; of 540.20
4. Find 125% of 356; of 276; of 540.20
5. Find 200% of 356; of 276; of 540.20
6. Find 150% of 356; of 276; of 540.20
7. Find 250% of 356; of 276; of 540.20

(a) Find the sum of the twenty-one results.

8. David's money is equal to 150% of Henry's money. Henry has \$240. How much has David?

9. If goods cost \$260, and the profit on them is 125%, what is the selling price?

10. If a Chicago lot at the beginning of a certain year was worth \$8000 and during the year increased in value 50%, what was it worth at the end of the year? What would it have been worth at the end of the year had it increased 250%?

Percentage.

223. PERCENTAGE PROBLEMS UNDER CASE II, IN WHICH THE PER CENT IS MORE THAN 100.

$1107.05 is 175% of how much money?

Operation No. 1.

175)$1107.05(6.326
1050 100
570 $632.60
525
455
350
1050
1050

Explanation.

Since $1107.05 is 175 hundredths of the money, 1 hundredth of the money is one 175th of $1107.05 or $6.326; and 100 hundredths, 100 times $6.326 or $632.60.

Operation No 2.

1.75)$1107.05(632.60
1050
570
525
455
350
1050
1050

Explanation.

We may find one hundred 175ths of a number by finding one 175th of 100 times the number. One hundred times $1107.05 is $110705. One 175th of $110705 is $632.60.

Operation and Explanation No. 3.

$175\% = \frac{175}{100} = \frac{7}{4}$. If $1107.05 is 7 fourths of the money, 1 seventh of $1107.05, or $158.15, is 1 fourth of the money, and 4 fourths, or the whole of it, is 4 times $158.15, or $632.60.

PROBLEMS.

1. 43.50 is 125% of what number?
2. 65.10 is 150% of what number?
3. 48.50 is 200% of what number?
4. 59.20 is 250% of what number?
5. 29.44 is 115% of what number?

(a) Find the sum of the five answers.

Percentage.

224. Percentage Problems under Case III, in which the per cent is more than 100.

$1107.05 is what per cent of $632.60? *

Operation.

$6.326)$1107.050(175
6326
47445
44282
31630
31630

Explanation.

One per cent of $632.60 is $6.326; then $1107.05 is as many per cent of $632.60 as $6.326 is contained times in $1107.05. It is contained 175 times, so $1107.05 is 175% of $632.60.

Note.—The above problem may be solved as a similar problem is solved on page 125, Operation No. 1. $1107.05 is $\frac{1107.05}{632.60}$ of $632.60. Perform the division indicated carrying out to hundredths only. The quotient will be 1.75 or $\frac{175}{100} = 175\%$.

Problems.

1. What per cent of 845 is 2112.5?
2. What per cent of 845 is 1056.25?
3. What per cent of 845 is 1352?
4. What per cent of 845 is 1183?
5. What per cent of 845 is 2746.25?

(a) Find the sum of the 5 answers.

6. Reuben has $2420; Bernie has $4961. Bernie's money equals how many per cent (hundredths) of Reuben's money?

7. A certain house cost $6425. The lot upon which it stands cost $2325. (a) The cost of the house equals how many per cent (hundredths) of the cost of the lot? (b) The cost of the lot equals how many per cent (hundredths) of the cost of the house?

* See "Operation and Explanation, No. 4," page 125.

Percentage.

225. Miscellaneous Problems.

1. Find $\frac{1}{2}$ %* of 632; of 356; of 272.
2. Find $\frac{1}{4}$ % of 632; of 356; of 272.
3. Find .25 %† of 632; of 356; of 272.

(a) Find the sum of the nine results.

4. Find $3\frac{1}{2}$ % of 496; of 532; of 720.
5. Find $4\frac{3}{4}$ % of 496; of 532; of 720.
6. Find 1.75 % of 496; of 532; of 720.

(b) Find the sum of the nine results.

7. What part of 94 is 11?‡ What per cent?
8. What part of 94 is 36? What per cent?
9. What part of 94 is 47? What per cent?

(c) Find the sum of the three "per cents."

10. 15 is 3 % of what number? 4 %? 5 %?
11. 24 is 3 % of what number? 4 %? 5 %?
12. 42 is 3 % of what number? 4 %? 5 %?

(d) Find the sum of the nine answers.

13. 375 is 125 % ($\frac{125}{100}$) of what number?
14. 436 is 109 % ($\frac{109}{100}$) of what number?
15. 598 is 115 % ($\frac{115}{100}$) of what number?

(e) Find the sum of the three answers.

16. 544 is 15 % less than what number?§
17. 545 is 25 % more than what number?
18. 510 is 170 % of what number?

(f) Find the sum of the three answers.

* This means $\frac{1}{2}$ of 1 per cent. † This means find 25 hundredths of 1 per cent.
‡ Answer with a common fraction in its lowest terms.
§ If it is 15 per cent less than the number it is 85 per cent of the number.

Algebra.

226. TO FIND TWO NUMBERS WHEN THEIR SUM AND DIFFERENCE ARE GIVEN.

1. The difference of two numbers is 4 and their sum is 20. What are the numbers?

Let x = the smaller number,
then $x + 4$ = the larger number,
and $x + x + 4 = 20$.
Transposing $x + x = 20 - 4$.
Uniting $2x = 16$.
Dividing $x = 8$, the smaller number
$x + 4 = 12$, the larger number.

2. The difference of two numbers is 9 and their sum is 119. What are the numbers?

3. The difference of two fractions is $\frac{9}{20}$ and their sum is $\frac{3}{4}$. What are the fractions?

4. The difference of two numbers is d and their sum is s. What are the numbers?

Let x = the smaller number
then $x + d$ = the larger number,
and $x + x + d = s$
Transposing $x + x = s - d$
Uniting $2x = s - d$
Dividing $x = \frac{s - d}{2}$

Observe that any number you please may be put in the place of s; and any number less than s, in the place of d: so when the sum and the difference of two numbers are given the smaller number may be found by subtracting the difference from the sum and dividing the remainder by 2.

5. The difference of two numbers is 327, and their sum is 1159. What are the numbers?

Algebra.

227. Another Method of Finding Two Numbers when their Sum and Difference are Given.

1. The difference of two numbers is 17 and their sum is 69. What are the numbers?

Let x = the larger number,
then $x - 17$ = the smaller number,
and $x + x - 17 = 69$.
Transposing $x + x = 69 + 17$.
Uniting $2x = 86$.
Dividing $x = 43$, the larger number.
$x - 17 = 26$, the smaller number.

2. The difference of two numbers is 8.4 and their sum 75.6. What are the numbers?

3. The difference of two numbers is d and their sum is s. What are the numbers?

Let x = the larger number,
then $x - d$ = the smaller number,
and $x + x - d = s$
Transposing $x + x = s + d$
Uniting $2x = s + d$
Dividing $x = \frac{s + d}{2}$

Observe that any number you please may be put in the place of s and any number less than s in the place of d; so when the sum and difference of two numbers are given the larger may be found by adding the difference to the sum and dividing the amount by 2.

4. A horse and a harness together are worth $146, and the horse is worth $74 more than the harness. Find the value of each.

Geometry.

228. How Many Degrees in Each Angle of a Regular Pentagon?

Fig. 1.

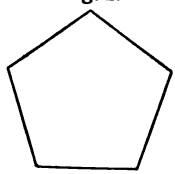
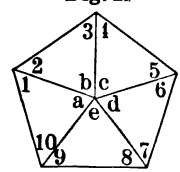


Fig. 2.



1. Every regular pentagon may be divided into — equal, isosceles triangles.

2. The sum of the angles of one triangle is equal to — right angles; * then the sum of the angles of 5 triangles is equal to — right angles.

3. But the sum of the central angles in figure 2, $(a+b+c+d+e)$ is equal to — right angles; † then the sum of all the other angles of the five triangles is equal to 10 right angles, less 4 right angles, or 6 right angles = 540°. But the angular space that measures 540°, as shown in Figure 2, is made up of 10 equal angles, so each one of the angles is 1 tenth of 540° or 54°. Two of these angles, as 1 and 2, make one of the angles of the pentagon; therefore each angle of the pentagon measures 2 times 54° or 108°.

4. Using the protractor construct a regular pentagon as follows:

(a) Draw two lines that meet in a point, each line being 2 inches long and the angular space between them being 108°.

(b) Regarding the two lines as two sides of a regular pentagon, draw two more sides each 2 inches long and joining those already drawn at an angle of 108°.

(c) Complete the figure by drawing the fifth side and prove your work by measuring the last line drawn and the other two angles of the pentagon.

* See page 59, 6 and 7. † See page 29, Art. 66.

229. Miscellaneous Review.

Remembering that in speaking of the per cent of loss or gain, the *cost is the base* unless otherwise specified, tell the per cent of loss or gain in each of the following:

1. Bought for 2 and sold for 3.
2. Bought for 3 and sold for 2.
3. Bought for 4 and sold for 5.
4. Bought for 5 and sold for 4.
5. Bought for 5 and sold for 6.
6. Bought for 6 and sold for 5.
7. Bought for 8 and sold for 10; for 12.
8. Bought for 8 and sold for 14; for 16.
9. Bought for 8 and sold for 18; for 20.
10. Bought for 8 and sold for 4; for 2.

11. Mr. Parker sold goods at a profit of 25 %; the amount of his sales on a certain day was $24.60. How much was his profit? *

12. Mr. Jewell sold goods at a loss of 25 %; the amount of his sales on a certain day was $24.60. How much was his loss? *

13. By selling a horse for $156 there was a loss to the seller of 20 %. What would have been his gain per cent if he had sold the horse for $234? †

14. A bill was made for wood that was supposed to be 4 feet long. It was afterwards found to be only 46 inches long. What % should be deducted from the bill?

15. The marked price on a pair of boots was 25 % above cost. If the dealer sells them for 25 % less than the marked price will he receive more or less than the cost of the boots?

16. If by selling goods at a profit of 12 % a man gains $6.60, what was the cost of the goods?

* $24.60 is what per cent of the cost?

† First find the cost of the horse.

PERCENTAGE.

230. Discounting Bills.

Many kinds of goods are usually sold "on time;" that is, the buyer may have 30, 60, or 90 days in which to pay for them. If he pays for such goods at the time of purchase or within ten days from the time of purchase, his bill is "discounted" from 1 % to 6 %, according to agreement; that is, a certain part of the amount of the bill is deducted from the amount.

Example.

Mr. Smith bought of Marshall Field & Co. a bill of goods amounting to $350.20. The discount for immediate payment ("spot cash") was 1 %. How much must he pay for the goods?

1 % of $350.20 is $3.50. $350.20 − $3.50 = $346.70.

Problems.

"Figure the discounts" on the following bills: †

1. Bill of $324.37, discounted at 2 %.
2. Bill of $276.45, discounted at 1 %.
3. Bill of $356.50, discounted at 3 %.
4. Bill of $536.50, discounted at 6 %.
5. Bill of $561.80, discounted at 4 %.

(a) Find the sum of the five bills before they are discounted.

(b) Find the sum of the discounts.

(c) Find the sum of the five bills after discounting.

† Fractions of cents in the discount on each bill may be ignored.

Applications of Percentage.

231. Discounts from List Price.

Dealers in hardware, rubber boots and shoes, belting, rubber hose, and many other kinds of goods, sell from a list price agreed upon by the manufacturers. The actual price is usually less than the list price. "20% off" means that the list price is to be discounted 20%. "20 and 10 off" means that the list price is to be discounted 20%, and what remains is to be discounted 10%. Sometimes as many as nine successive discounts are allowed. Observe that in computing these the base changes with each discount.

Problems.

Find the actual cost of—

1. 500 ft. $\frac{3}{4}$-inch gas pipe, (list, 7¢ per ft.) at 50 and 10 off.
2. 350 ft. $\frac{1}{2}$-inch gas pipe, (list, 8¢ per ft.) at 50 and 10 off.
3. 200 ft. $1\frac{1}{2}$-inch gas pipe (list, 26¢ per ft.) at 55 and 10 off.
4. 260 ft. 2-inch gas pipe (list, 35¢ per ft.) at 55 and 10 off.
5. 48 $\frac{1}{2}$-in. elbows, (list, 7¢ each) at 65 and 20 off.
6. 36 $\frac{3}{4}$-in. elbows, (list, 9¢ each) at 65 and 20 off.

(a) Find the entire cost of the six items.

7. Find the cost of 12 pairs men's rubber boots, (list, $3.00 per pair) at 25 and 10 off. Find the cost of the same at 35 off. Why are the results unlike?
8. Which is the lower price, 50 and 10 off, or 60 off, the list being the same?
9. Bought for 40 off from list price and sold for 10 off from list price. What was my gain per cent?
10. Bought for 70 off from list and sold for 50 and 20 off from list. Did I lose or gain and how many per cent?

Applications of Percentage.

232. SELLING "on Commission."

When goods are sold "on commission" *the selling-price is the base;* that is, the seller receives a certain per cent of the selling price as remuneration for services.

Commission is the sum paid an agent, or commission merchant, for transacting business.

PROBLEMS.

1. At 40 %, what is the commission for selling $275 worth of books? If the salesman sells and collects for 40% of the selling price, how much of the $275 will he retain and how much "pay over" to the man for whom he sells the books?

2. While selling books on a commission of 40% my commission amounted to $56. What was the selling price of the books? If I not only sold but collected for 40% of the selling price, how much money should I "pay over" to my employer?

3. A real estate agent sold a house and lot for $4250. If his commission is 5%, how much should he receive for his services?

4. A real estate agent sold a piece of property upon which his commission at 5% amounted to $275. What was the selling price of the property? How much should the owner receive for the property after deducting the commission?

5. A commission merchant sold 2140 lb. of butter at 23¢ a pound. After deducting his commission of 5% and paying freight charges of $36.50, and storage charges of $21.40, how much should he send to the man for whom he made the sale?

Applications of Percentage.

233. Taxes.

A **tax** is a sum of money paid for public purposes. A tax on property is reckoned at a certain per cent of the **assessed value** of the property. The assessed value may or may not be the real value. It is often much below the real value.

PROBLEMS.

1. Mr. Hardy has a farm of 240 acres which he values at $24000. Its assessed value is $22 per acre. If his state tax is $\frac{1}{2}\%$, his county tax $1\frac{1}{4}\%$, his town tax $\frac{1}{4}\%$, his school tax 2%, and his special road and bridge tax 1%, how much money must he pay as taxes on his farm?

2. The assessed value of the taxable property of a certain school district is $176,242.25. If the school tax is $2\frac{1}{2}\%$ and the collector receives 2% of the amount collected as his commission, and collects the entire amount of the tax, how much should the district officers receive from this source for school purposes?

3. The assessed value of Mr. Randall's property is $3400. At the rate of 15 mills on a dollar*, how much tax must he pay?

4. The assessed value of the property of a district of a certain city is $250,000. (a) What must be the per cent of taxation to raise $10,000? (b) What will be the net sum realized for public purposes if the collector is able to collect only 95% of this tax and he receives for his services 2% of the amount collected?

5. Mr. Evan's tax is $35.60; the rate of taxation is $2\frac{1}{2}\%$. What is the assessed value of his property? †

* "15 mills on a dollar" is the same as $1\frac{1}{2}\%$.

† $35.60 is what part of the assessed value?

Applications of Percentage.

234. Insurance is a guaranty by one party to pay a certain sum to another party in the event of loss or damage.

The **policy** is the written contract given by the underwriter to the insured. The **premium** is the sum paid for insurance.

PROBLEMS.

1. A store valued at $7500 was insured for $5000 for 1 year. The rate of insurance was 2%. What was the amount of the premium?

2. A stock of goods valued at $10000 was insured for $5000. A fire occurred, but part of the goods were saved. It was found that the entire loss to the owner of the goods was $4750. (a) How much should he receive? (b) How much should he receive if the loss were $5750?*

3. An insurance agent offers to insure my farm buildings for $3500 for 1 year at 1%, or for 5 years at 3%; the entire premium in either case to be paid in advance. (a) If I accept the first proposition, how much is the premium to be paid? (b) How much if I accept the second?

4. What is the rate of insurance on the nearest store and stock of goods? On farm property? On village or city property other than stores?†

5. A large building was insured in one company for $25000, in another company for $15000. It was damaged by fire to the extent of $12800. How much of the damage should each company pay?

NOTE.—The companies must share the loss in proportion to the amount of insurance carried by them.

*In case of total loss the owner would receive $5000. In case of partial loss the owner should receive the full amount of the loss, provided it does not exceed $5000.

† Any insurance agent will be willing to answer these questions for you.

235. Miscellaneous Problems in Applications of Percentage.

1. A dealer who had marked goods 50 % above the cost, sold them after deducting 10 % from the marked price. His profit on that sale was what per cent of the cost of the goods?

2. By selling a suit of clothes for $7.20 I would lose an amount equal to 10 % of the cost. For what must I sell the suit to gain a sum equal to 10 % of the cost?

3. I sold goods at a loss of 7 %. My actual loss was $3.50. What was the cost of the goods?

4. The real value of a stock of goods was $8250. They were insured for $5500. A fire occurred and the salvage amounted to only $575. If the insurance company promptly settles in accordance with the above facts what is my actual loss by the fire?

5. If I sell goods on a commission of 12½ %, what must be the amount of my sales in order that I may receive an annual salary of $2500?

6. A school numbers 140 pupils. The absence for one week was as follows: Monday, 3 days; Tues., 5 days; Wed., 4 days; Thurs., 5 days; Friday, 3 days. (a) What was the per cent of absence? (b) What was the per cent of attendance?

7. Sold a horse for $120 and gained 25 %. What did the horse cost me?

8. When the cost is ⅘ of the selling price what is the gain per cent?

9. When the selling price is ⅘ of the cost what is the loss per cent?

10. The sum collected was $2321.50; commission for collecting was 6 %. What was the cost of collecting?

Algebra.

236. Miscellaneous Problems.

1. In a school there are 896 pupils. There are three times as many boys as girls. How many girls? How many boys?

2. A man had 235 sheep. In the second flock there were 15 more sheep than in the first. In the third flock there were 20 fewer than in the first. How many sheep in each flock?

Note.—Let x = the number in the first flock; then $x + 15$ = the number in the second, and $x - 20$, the number in the third.

3. A man owns three farms. In the second there are half as many acres as in the first. In the third there are twice as many acres as in the first. In all there are 560 acres. How many acres in each farm?

4. In an apple and pear orchard containing 296 trees, there were 5 more than twice as many apple trees as pear trees. How many of each kind?

5. To a number I add one half of itself and 15 and have 150. What is the number?

6. From three times a number I subtract $\frac{2}{3}$ of the number and 5, and have 37 remaining. What is the number?

Note.—Let x = the number; then $3x - \left(\frac{2x}{3} + 5\right) = 37$. On removing the parenthesis, what sign must be changed? See page 67, II.

7. If to three times a number I add $\frac{2}{3}$ of the number and 18, the sum will be 238. What is the number?

8. Two thirds of a number is equal to the number decreased by 56. What is the number?

$$\frac{2x}{3} = x - 56$$

Algebra.

237. Miscellaneous Problems.

1. A is 50 years old. B is 20 years old. In how many years will A be only twice as old as B?

Note.—Let x = the number of years; then $(20 + x) \times 2 = 50 + x$.

2. Find four consecutive numbers whose sum is 150.

Note.—Let x = the first; then $x + 1$ = the second; $x + 2$ = the third, etc

3. Find three consecutive numbers whose sum is 87.

4. Two numbers have the same ratio as 2 and 3, and their sum is 360. What are the numbers?

Note.—Let $2x$ = the first, and $3x$ = the second.

5. Two numbers have the same ratio as 3 and 4, and their sum is 168. What are the numbers?

6. Two numbers have the same ratio as 2 and 5, and their difference is 87. What are the numbers?

7. A has $350. B has $220. How many dollars must A give to B so that each may have the same sum?

Note.—Let x = the number of dollars that must be given by A to B; then $220 + x = 350 - x$.

8. C has $560. D has $340. How many dollars must C give to D so that each may have the same sum?

9. E has $630. F has $240. How many dollars must E give to F so that E will have exactly twice as many dollars as F?

10. The fourth and the fifth of a certain number are together equal to 279. What is the number?

11. The difference between 1 fourth and 1 fifth of a certain number is 28. What is the number?

Geometry.

238. How many Degrees in each Angle of a Regular Hexagon?

Fig. 1.

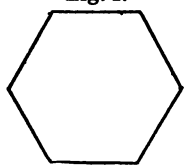
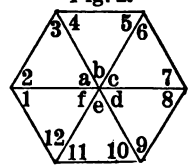


Fig. 2.



1. Every regular hexagon may be divided into —— equal isosceles triangles.

2. The sum of the angles of one triangle is equal to —— right angles,* then the sum of the angles of 6 triangles is equal to —— right angles.

3. But the sum of the central angles in Fig. 2 ($a + b + c + d + e + f$) is equal to —— right angles; † then the sum of all the other angles of the six triangles is equal to 12 right angles less 4 right angles or 8 right angles = 720°. But the angular space that measures 720°, as shown in Fig. 2, is made up of 12 equal angles; so each one of the angles is one 12th of 720° or 60°. Two of these angles, as 1 and 2, make one of the angles of the hexagon; therefore each angle of the hexagon measures 2 times 60° or 120°.

4. Using the protractor, construct a regular hexagon, making each side 2 inches long.

Observe that since all the angular space about a point is equal to 4 right angles, or 360°, and since the space around the central point of the hexagon is divided into 6 equal angles, each of these angles is an angle of (360° ÷ 6), 60°. But each of the other angles of these triangles has been shown to be an angle of 60°; so each triangle is equiangular. Are the triangles equilateral?

*See page 59, 6 and 7. †See page 29, Art. 66.

239. Miscellaneous Review.

1. A man buys goods for $60 and sells them for $75. He gains ——— dollars.

(a) The gain equals what part of the cost? What %?

(b) The gain equals what part of the selling price? What per cent?

(c) The cost equals what part of the selling price? What per cent?

2. When the cost is 2 thirds of the selling price what is the per cent of gain?

3. When the selling price is 2 thirds of the cost what is the per cent of loss?

4. Bought for $200 and sold for $300. What was the per cent of gain?

5. Bought for $300 and sold for $200. What was the per cent of loss?

6. A tax of 15 mills on a dollar was levied in a certain town, the assessed value of the taxable property being $475,250. If 5 % of the tax proves to be non-collectable and if the collector is allowed 2 % of the amount collected, for his services, how much will be realized from the levy?

7. Which is the greater discount, "20 and 10 and 5 off" or "35 off"?

8. A sold goods for B on a commission of 15 %. His sales for a certain period amounted to $780. If the goods cost B exactly $600 was B's net profit more or less than 10 %?

9. A offers rubber boots at "50 and 20 off"; B offers them at "20 and 50 off". The quality and list price being the same, which offer shall I accept?

PART II.

INTEREST.

240. Interest is compensation for the use of money.

241. The money for which interest is paid is called the **principal.**

242. The principal and interest together are called the **amount.**

NOTE.—Interest is usually reckoned in per cent, the principal being the base; that is, the borrower pays for the use of money a sum equal to a certain per cent of the principal. "The rate of interest" is the per cent per annum which the borrower agrees to pay. When a man loans money "at 6%" he expects to receive back the principal, and a sum equal to 6 % of the principal for every year the money is loaned and at that rate for fractions of years.

EXAMPLE.

Find the interest of $257 for two years at 6 %.

Operation.	*Explanation.*
$2\overset{v}{5}7$ .12 5.14 25.7 $30.84	The interest of any sum for 2 years at 6 % is 12 hundredths of the principal. One hundredth of $257 is $2.57, and 12 hundredths of $257 is 12 times $257 or $30.84.

1. Find the interest of $242 for 3 yr. at 7 %.
2. Find the interest of $375 for 2 yr. at 6 %.
3. Find the interest of $146 for 1 yr. at 5 %.
4. Find the interest of $274 for 3 yr. at 5 %.
5. Find the interest of $375 for 2 yr. at 8 %.
6. Find the interest of $864 for 3 yr. at 7 %.

(a) Find the sum of the six results.

Interest.

243. To Compute Interest for any Number of Years and Months.

Note.—The interest for 1 month is 1 twelfth as much as it is for 1 year; for 2 months, 2 twelfths or 1 sixth, etc.

Example.

Find the interest of \$324.50 for 2 yr. 5 mo. at 6%.

Operation and Explanation No. 1.

Interest of \$324.50 for 1 year at 1% = \$3.245	
Interest of \$324.50 for 1 year at 6% = \$19.470	
Interest of \$324.50 for 2 years at 6% =	\$38.940
Interest of \$324.50 for 1 mo. at 6% = \$1.6225	
Interest of \$324.50 for 5 mo. at 6% =	8.1125
Interest of \$324.50 for 2 yr. 5 mo. at 6% =	\$47.0525

Operation No. 2.

2 yr. 5 mo. = $2\frac{5}{12}$ years.

$2\frac{5}{12}$ times .06 = $.14\frac{1}{2}$

\$324.50
.14½
1.6225
12.9800
32.450
\$47.0525

Explanation.

The interest of any sum for 2 yr. 5 mo. is 14½ hundredths of the principal.

1 hundredth of \$324.50 is	\$3.2450
½ hundredth of \$324.50 is	\$1.6225
4 hundredths of \$324.50 is	12.9800
10 hundredths of \$324.50 is	32.450
14½ hundredths of \$324.50 is	\$47.0525

Problems.

1. Find the interest of \$325.40 for 1 yr. 6 mo. at 7%.*
2. Find the interest of \$420.38 for 2 yr. 10 mo. at 6%.
3. Find the interest of \$221.60 for 2 yr. 3 mo. at 6%.
4. Find the interest of \$145.20 for 1 yr. 9 mo. at 5%.
5. Find the interest of \$340.10 for 3 yr. 1 mo. at 4%.

(a) Find the sum of the five results.

* In an interest problem, if there is a fraction of a cent in the answer and this fraction is not greater than ½, it may be disregarded; if it is more than ½, regard it as 1 cent.

Interest.

244. To Compute Interest for any Number of Years, Months and Days.

Note.—In computing interest, 30 days is usually regarded as 1 month, and 360 days as 1 year; so each day is $\frac{1}{30}$ of a month or $\frac{1}{360}$ of a year.

Example.

Find the interest of \$256.20 for 2 yr. 7 mo. 13 days at 6 %.

Operation and Explanation No. 1.

Interest of \$256.20 for 1 yr.	at 1 % =	\$2.5620	
Interest of \$256.20 for 1 yr.	at 6 % =	\$15.3720	
Interest of \$256.20 for 2 yr.	at 6 % =		\$30.7440
Interest of \$256.20 for 1 mo.	at 6 % =	\$1.2810	
Interest of \$256.20 for 7 mo.	at 6 % =		\$8.9670
Interest of \$256.20 for 1 da.	at 6 % =	\$.04270	
Interest of \$256.20 for 13 da.	at 6 % =		.5551
Interest of \$256.20 for 2 yr. 7 mo. 13 da.	at 6 % =		\$40.2661

Operation No. 2.

2 yr. 7 mo. 13 da. $2\frac{13}{360}$ yr.

$2\frac{13}{360}$ times .06 = $.15\frac{13}{60}$

\$256.20
.15 13/60
1.8361
12.8100
25.620
\$40.2661

Explanation.

The interest of any sum for 2 yr. 7 mo. 13 da. at 6 % is $.15\frac{13}{60}$ of the principal.

1 hundredth of \$256.20 is	\$2.5620
$\frac{13}{60}$ of 1 hundredth of \$256.20 is	\$1.8361
5 hundredths of \$256.20 is	\$12.81
10 hundredths of \$256.20 is	\$25.62
$15\frac{13}{60}$ hundredths of \$256.20 is	\$40.2661

Problems.

1. Find the interest of \$350.40 for 2 yr. 5 mo. 7 da. at 6 %.
2. Find the interest of \$145.30 for 1 yr. 7 mo. 10 da. at 8 %.
3. Find the interest of \$174.20 for 2 yr. 3 mo. 15 da. at 7 %.
4. Find the interest of \$253.40 for 3 yr. 6 mo. 20 da. at 5 %.

(a) Find the sum of the four results.

245. The "six-per-cent method."

NOTE.—If the teacher so prefers, the problems on the three preceding pages, as well as those that follow, may be solved by the "six per cent method."

EXPLANATORY.

The interest at 6% for 1 yr. = .06 of the principal.
The interest at 6% for 1 mo. = $\frac{1}{12}$ of .06, or .005 of the principal.
The interest at 6% for 1 day = $\frac{1}{30}$ of .005 or .000$\frac{1}{6}$ of the principal.

READING EXERCISE.

1. Interest for 1 yr. at 6% = ——100ths of the principal;

for 2yr. = ——100ths; for 3 yr. = ——100ths;
for 1mo. = ——1000ths; for 2 mo. = ——1000ths;
for 3mo. = ——1000ths; for 4 mo. = ——1000ths;
for 5mo. = ——1000ths; for 6 mo. = ——1000ths;
for 1 da. = ——1000th; for 3 da. = ——1000th;
for 6 da. = ——1000th; for 12 da. = ——1000ths;
for 18 da. = ——1000ths; for 24 da. = ——1000ths;
for 25 da. = ——1000ths; for 27 da. = ——1000ths;

Find the interest of $243.25 for 2 yr. 5 mo. 18 da. at 6%.

OPERATION AND EXPLANATION.

Interest for 2 yr. =	.12 of the principal.
Interest for 5 mo. =	.025 of the principal.
Interest for 18 da. =	.003 of the principal.
Total interest =	.148 of the principal.

$243.25 × .148 = $36.001*

*Nota Bene.

(1) $36.001 plus $\frac{1}{6}$ of $36.001 = int. of same prin. for the same time at 7%.

(2) $36.001 less $\frac{1}{6}$ of $36.001 = int. of same prin. for the same time at 5%.

(3) How find the interest at 8%? at 9%? at 4%? at 3%?

Interest.

246. PROBLEMS TO BE SOLVED BY THE "SIX PER CENT METHOD."

1. Find the interest of \$265 for 1 yr. 3 mo. 13 da. at 6 %.*
2. Find the interest of \$346 for 2 yr. 5 mo. 20 da. at 6 %.†
3. Find the interest of \$537 for 1 yr. 7 mo. 10 da. at 6 %.‡
4. Find the interest of \$428 for 3 yr. 3 mo. 14 da. at 6 %.
5. Find the interest of \$150 for 1 yr. 6 mo. 15 da. at 6 %.

(a) Find the sum of the five results.

6. Find the interest of \$245.30 for 6 mo. 18 da. at 7 %.§
7. Find the interest of \$136.25 for 8 mo. 10 da. at 7 %.
8. Find the interest of \$321.42 for 5 mo. 15 da. at 7 %.
9. Find the interest of \$108.00 for 10 mo. 8 da. at 7 %.
10. Find the interest of \$210.80 for 7 mo. 21 da. at 7 %.

(b) Find the sum of the five results.

11. Find the amount of \$56.25 for 2 yr. 4 mo. 2 da. at 6 %.
12. Find the amount of \$31.48 for 1 yr. 5 mo. 11 da. at 6 %.
13. Find the amount of \$55.36 for 2 yr. 8 mo. 12 da. at 6 %.
14. Find the amount of \$82.75 for 2 yr. 10 mo. 8 da. at 6 %.
15. Find the amount of \$27.35 for 1 yr. 1 mo. 1 da. at 6 %.

(c) Find the sum of the five results.

16. Find the amount of \$875 for 3 yr. 8 mo. 15 da. at 5 %.
17. Find the amount of \$346 for 2 yr. 6 mo. 12 da. at 4 %.
18. Find the amount of \$500 for 1 yr. 7 mo. 18 da. at 3 %.
19. Find the amount of \$600 for 2 yr. 5 mo. 21 da. at 8 %.
20. Find the amount of \$825 for 1 yr. 9 mo. 24 da. at 9 %.

(d) Find the sum of the five results.

*1 yr.	.06	†2 yr.	.12	‡1 yr.	06.
3 mo.	.015	5 mo.	.025	7 mo.	.035
13 da.	.002½	20 da.	.003⅓	10 da.	.001⅔
Total	.077½	Total	.148⅓	Total	.096⅓

§ First find the interest at 6 %; then add ⅛ of that sum to itself.

247. To Find the Time Between two Dates.

EXAMPLE.

How many years, months, and days from Sept. 25, 1892, to June 10, 1896?

THE USUAL METHOD.

1896 - 6 - 10	From the 1896th yr., the 6th mo., and the 10th day, subtract the 1892nd yr., the 9th mo., and the 25th day. Regard a month as 30 days.
1892 - 9 - 25	
3 - 8 - 15	

A BETTER METHOD.

From Sept. 25, 1892 to Sept. 25, 1895, is 3 years.
From Sept. 25, 1895 to May 25, 1896, is 8 months.
From May 25, 1896 to June 10, 1896, is 16 days.

NOTE.—The two methods will not always produce the same results. The greatest possible variation is two days. Find the time from Jan. 22, 1895, to March, 10, 1897, by each method, and compare results. The difference arises from the fact that the month as a measure of time is a variable unit—sometimes 28 days, sometimes 31. The "*usual method*" regards each month as 30 days; the "*better method*" counts first the whole years, then the whole months, then the days remaining. By the "usual method," the time from Feb. 28, 1897 to March 1, 1897, is 3 days; by the "better method," it is 1 day.

PROBLEMS.

Find the time by both methods and compare the results.

1. From March 15, 1894 to Sept. 10, 1897.
2. From March 15, 1894 to Sept. 20, 1897.
3. From May 25, 1895 to Sept. 4, 1898.
4. From May 25, 1895 to Oct. 4, 1898.
5. From June 28, 1894 to Mch. 1, 1898.
6. From June 28, 1894 to May 1, 1898.
7. From Jan. 10, 1892 to Jan. 25, 1898.
8. From Jan. 10, 1892 to Dec. 25, 1898.
9. From April 15, 1893 to Aug. 15, 1898.

248. Algebra Applied to Problems in Percentage.

Example.

75 is 15 per cent of what number?

Let x = the number sought.

Then $\frac{15}{100}$ of x, or $\frac{15x}{100} = 75.$

Multiplying by 100, $15x = 7500.$

Dividing by 15, $x = 500.$ Ans.

Problems.

1. 56 is 2 per cent of what number?
2. 45 is 5 per cent of what number?
3. 60 is 12 per cent of what number?
4. 37 is 4 per cent of what number?
5. 53 is 8 per cent of what number?
6. n is r per cent of what number?

Let x = the number sought.

Then $\frac{rx}{100} = n$

Multiplying by 100, $rx = 100n$

Dividing by r, $x = \frac{100n}{r}$*

7. James has $54.20, and James's money equals 40 per cent of Henry's money. How much money has Henry?

8. Mr. Williams's annual expenses are $791.20; this is 92 per cent of his annual income. How much is his annual income?

9. Mr. Randall has 450 sheep; these equal 125 per cent of Mr. Evans's sheep. How many sheep has Mr. Evans?

* To the Teacher.—Encourage pupils to solve the 7th, 8th, and 9th problems in three ways: 1st, by the method given above; 2nd, arithmetically (Prob. 7—Henry's money equals 100 40ths of James's money); 3rd, by the application of the formula, $\frac{100n}{r} = x$.

249. Algebra Applied to Problems in Percentage.

Example.

60 is what per cent of 75?

Let x = the per cent (number of hundredths).

Then $\frac{x}{100}$ of 75, or $\frac{75x}{100} = 60.$

Multiplying by 100, $75x = 6000$

Dividing by 75, $x = 80.$ Ans.

Problems.

1. 180 is what per cent of 200?
2. 17 is what per cent of 340?
3. \$87.50 is what per cent of \$250?
4. 81 is what per cent of 540?
5. \$75.60 is what per cent of \$630?
6. n is what per cent of b? *

Let x = the per cent.

Then $\frac{x}{100}$ of b or, $\frac{bx}{100} = n.$

Multiplying by 100, $bx = 100n$

Dividing by b, $x = \frac{100n}{b}$ *

To the Teacher.—Require the pupils to solve the first five problems in four ways:

(1) Let x = the per cent and solve as the "example" is solved.

(2) Using one hundredth of each base as a divisor and the other number mentioned in the problem as a dividend, find the quotient.

(3) Find what part the first number mentioned in each problem is of the base, and change the fraction thus obtained to hundredths.

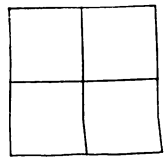
(4) Apply the formula, $\frac{100n}{b} = x.$

* The b in problem 6, and in the formula may be thought of as standing for the base.

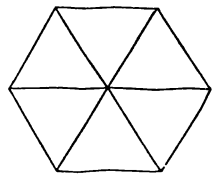
Geometry.

250. SOME INTERESTING FACTS ABOUT SQUARES, TRIANGLES, AND HEXAGONS.

1. Four equal squares may be so joined as to cover all the space about a point. Each angle whose vertex is at the central point of the figure is an angle of 90°. Four times 90° = ——— degrees.

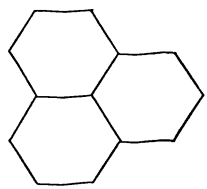


2. Six equal equilateral triangles may be so joined as to cover all the space about a point. Each angle whose vertex is at the central point of the figure is an angle of 60°. Six times 60° = ——— degrees.



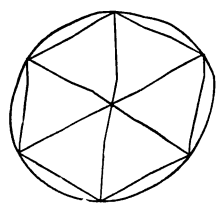
NOTE.—Cut from paper 6 equal equilateral triangles and join them as shown in the figure.

3. Three equal hexagons may be so joined as to cover all the space about a point. Each angle whose vertex is at the central point of the figure is an angle of 120°. Three times 120° = ——— degrees.



NOTE.—Cut from paper 3 equal hexagons and join them as shown in the figure.

4. Since every regular hexagon may be divided into six equal equilateral triangles,* as shown in the figure, it follows that the side of a regular hexagon is exactly equal to the radius of the circle that circumscribes the hexagon.



*See *Observation*, page 149.

251. Miscellaneous Reviews.

1. Find the interest of $250 from Sept. 5, 1896, to Jan. 17, 1898, at 6%.

2. Find the amount of $340 from April 19, 1895, to Oct. 1, 1896, at 6%.

3. Find the amount of $500 from May 20, 1897, to Feb. 28, 1898, at 5%.

4. Find the amount of $630 from July 1, 1896, to Nov. 1, 1896, at 7%.

5. Find the amount of $800 from Jan. 1, 1897, to Jan. 25, 1897, at 6%.

6. If a 60-day bill of $400 is discounted 2% for cash, how much ready money will be required to pay the bill?

7. Find the amount of $392 for 60 days (two months) at 6%.

Note.—Observe that $392 is the answer to problem 6. The result of problem 7, then, is the amount that the goods mentioned in problem 6, would cost at the end of 60 days if the purchaser borrowed the money at 6% to pay for them. Compare this result with $400. How much does the purchaser of the bill save by borrowing the money at 6% to pay the bill instead of letting the bill run 60 days and then paying $400?

8. Find the cost of goods, the list price being $46, and the discounts "50 and 10 off and 2 off for cash."

9. My remuneration for selling goods on a commission of 40% amounted to $56.20. How much should the man for whom the goods were sold receive?

Note.—$56.20 is 40% of the selling price of the goods. The man for whom the goods were sold should receive 60% of the selling price When goods are sold on a commission of 40%, what the agent receives equals what part of what the employer receives? What the employer receives is how many times what the agent receives?

10. The sum of all the angles of 6 triangles equals how many right angles?

PROMISSORY NOTES.

252. When a man borrows money at a bank he gives his note for a specified sum to be paid at a specified time and he receives therefor, not the sum named in the note, but that sum less the interest upon it from the time the note is given to the bank officials to the time the note is due.

253. The acceptance of a note by the bank officials and the payment of a sum less than it will be worth at maturity* is called "*discounting the note.*"

254. The **proceeds** of a note is the sum paid for it. As a rule, the notes discounted at a bank are those which bear no interest, and the date of discounting is usually the same as the date of the note, though not necessarily so.

EXAMPLE.

Find the discount and proceeds of the following:

$800 JACKSONVILLE, ILL., Jan. 10, 1898.

Sixty days after date I promise to pay James Rice or order, eight hundred dollars, value received.

ARTHUR WILLIAMS.

Discounted at 6%, Jan. 10, 1898.

From the date of discount to the date of maturity it is 60 days. Interest of $800 for 60 days = $8.00.

Proceeds of note = $800 − $8 = $792.

Observe that the bank receives the interest on $800 for two months at 6% for the use of $792 for two months. The actual rate of interest is therefore a little more than the rate named.

* The date of maturity is the day upon which the note is legally due.

255. Problems in Bank Discount.

1. $375.00. Chicago, Ill., Apr. 5, 1898.

Thirty days after date I promise to pay to the order of John Smith, three hundred seventy-five dollars, at the Union National Bank. Value received.

James White.

Discounted April 5, at 6 %. Find proceeds.

2. $450. Aurora, Ill., March 10, 1898.

Sixty days after date I promise to pay to Wm. George, or order, four hundred fifty dollars, at the Old Second National Bank. Value received.

Fay D. Winslow.

Discounted March 10, at 7 %. Find proceeds.

3. $2300.00. Waukegan, Ill., Feb. 9, 1898.

Ninety days after date I promise to pay to the order of John Mulhall, two thousand three hundred dollars, at the Security Savings Bank. Value received.

Chas. Whitney.

Discounted Feb. 9, at 7 %. Find proceeds.

4. $5000.00. Serena, Ill., Jan. 20, 1898.

Sixty days after date, I promise to pay to John Parr, or order, five thousand dollars. Value received.

Harry Brown.

Discounted Jan. 20, at 7 %. Find proceeds.

5. $3500. Boston, Mass., April 12, 1898.

Sixty days after date I promise to pay to W. J. Button, or order, three thousand five hundred dollars. Value received. Harry Wilson.

Discounted Apr. 24, at 6 %. Find proceeds.

Promissory Notes.

256. The Discounting of Interest-Bearing Notes.

Whenever a note is presented at a bank to be discounted, *its value at maturity* is regarded as the sum upon which the discount is to be reckoned. When the note is not interest bearing, its face value* is its value at maturity. In the discounting of short-time notes at banks it is customary to find the exact number of days from the date of discounting to the date of maturity, and to regard each day as 1/360 of a year.

Example.

$750.00. Austin, Ill., Jan. 1, 1898.

Six months after date I promise to pay to the order of N. D. Gilbert, seven hundred fifty dollars, with interest at the rate of six per cent per annum. O. T. Bright.

Discounted at a bank, May 10, 1898, at 7 %.

Value of the note at maturity,	$772.50
Discount 52 days, at 7%,	7.81
Proceeds,	$764.69

1. $540.00. Topeka, Kansas, Dec. 10, 1897.

Five months after date, I promise to pay to the order of J. C. Thomas, five hundred forty dollars, with interest at the rate of six per cent per annum. Hiram Baker.

Discounted at a bank, April 1, 1898, at 7 %.

2. $325.00. Earlville, Ill., Feb. 1, 1898.

Six months after date, I promise to pay to the order of Wm. R. Haight, three hundred twenty-five dollars, with interest at the rate of six per cent per annum.

Chas. Hoss.

Discounted at a bank, June 1, 1898, at 7 %.

* The *face value*, or "face," of a note is the value shown upon its face—the sum of money expressed in words in the body of the note.

257. Partial Payments on Notes.

A **partial payment** is a part-payment made upon a note before the time of final settlement.

There are two methods in common use of finding the value of a note at maturity, upon which one or more partial payments have been made. The first method is often applied to computation of "short-time notes," such as run one year or less. A formal statement of this method is called the—

Merchants' Rule.—*Find the amount of the face of the note from the date to maturity. Then find the amount of each payment from the time it was made to the maturity of the note. From the amount of the face of the note subtract the sum of the amounts of the payments.*

$450. WAUKEGAN, ILL., Jan. 1, 1897.

One year after date I promise to pay to the order of Wm. E. Toll, four hundred fifty dollars, with interest at six per cent. Value received.

JEROME BIDDLECOM.

Part payments were made upon this note as follows: March 1, 1897, $250; May 1, 1897, $150.

Amount of the face of the note for one year,		$477.00
Amount of $250, March 1, '97 to Jan. 1, '98	$262.50	
Amount of $150, May 1, '97 to Jan 1, '98	156.00	$418.50
Value at maturity,		$ 58 50

1. $1000.00. SPRINGFIELD, ILL., Jan. 1, 1898.

Eight months after date I promise to pay to the order of Fred H. Wines, one thousand dollars, with interest at six per cent. Value received.

J. H. FREEMAN.

Part payments were made upon this note as follows: Apr. 1, 1898, $350; June 15, 1898, $240.

How much was due on this note at maturity, Sept. 1, 1898?

Promissory Notes.

258. A decision of the United States Supreme Court many years ago, led to the very general adoption in the solution of "partial payment problems," of what is now known as the United States Rule.—*Find the amount of the principal at the time of the first payment. Subtract the first payment from this amount. The remainder is a new principal, upon which find the amount at the time of the second payment. Subtract the second payment from this amount. Continue this process to the time of settlement. The last amount is the sum due.*

An exception to the foregoing rule is required when there is a payment which is less than the interest due at the time the payment is made. In such case the payment is treated as though made at the time of the next payment; and if the two payments together do not equal the interest due, the sum of both is again carried forward.

$950. PETERSBURG, ILL., Jan. 1, 1895.

On or before Jan. 1, 1898, I promise to pay to N. W. Branson, or order, nine hundred fifty dollars, with interest at six per cent. Value received. B. LANING.

Part payments were made on this note as follows: July 1, 1895, $150; Jan. 1, 1896, $200; Jan. 1, 1897, $250. Find the amount due Jan. 1, 1898.

Amount of $950, July 1, 1895 (6 mo.)	$978.50
Subtract first payment	150.00
New principal	$828.50
Amount of $828.50, Jan. 1, 1896 (6 mo.)	$853.35
Subtract second payment	200.00
New principal	$653.35
Amount of $653.35, Jan. 1, 1897 (1 year)	$692.55
Subtract third payment	250.00
New principal	$442.55
Amount of $442.55, Jan. 1, 1898 (1 year)	$469.10

259. PROBLEMS IN "PARTIAL PAYMENTS."

1. Date of note, Jan. 1, 1896.
Face of note, $800.
Rate of interest, six per cent.
Payment May 1, 1896, $125.
Payment Dec. 1, 1896, $230.
Find amount due Jan. 1, 1898.

2. Date of note, July 10, 1896.
Face of note, $500.
Rate of interest, six per cent.
Payment, Dec. 22, 1896, $200.
Payment, July 15, 1897, $200.
Find amount due Jan. 1, 1898.

3. Date of note, Jan. 1, 1897.
Face of note, $600.
Rate of interest, six per cent.
Payment Sept. 1, 1897, $100.
Find amount due Jan. 1, 1898.

NOTE.—Solve problem 3 by the Merchants' Rule and by the United States Rule. The answer obtained by the latter will be 48 cents larger than the answer obtained by the former.

Observe that the Merchants' Rule is based upon the supposition that interest is not due until the time of settlement of the note, while the U. S. Rule is based upon the supposition that interest is due whenever a payment is made. By applying the latter rule to problem 3, $24 of interest must be paid Sept. 1; by applying the former rule to the same problem the entire $100 paid Sept. 1, applies in payment of principal. The answer, then, by the U. S. Rule, must be greater than the answer by the Merchants' Rule, by the interest on $24 for four months, or 48 cents.

4. Change the interest in all the above problems to seven per cent, solve them by the U. S. Rule, and find the sum of the amounts due Jan. 1, 1898.

Algebra.

260. ALGEBRA APPLIED TO SOME PROBLEMS IN INTEREST.

EXAMPLE.

What principal at 6% will gain $96 in 2 yrs.?

Let x = the principal.

Since the interest at 6% for 2 years equals $\frac{12}{100}$ of the principal

then $\frac{12}{100}x$, or $\frac{12x}{100} = 96.$

Multiplying by 100 $\quad 12x = 9600$

$x = 800.$

PROBLEMS.

1. What principal at 6% will gain $67.50 in 2 years, 6 months?*

2. What principal at 6% will gain $27.20 in 1 year 4 months?

3. What principal at 7% will gain $87.50 in 2 years 6 months?

Let x = the principal; then $\frac{17\frac{1}{2}}{100}x$, or $\frac{35}{200}x = \$87.50.$

4. What principal at 5% will gain $187.50 in five years?

5. What principal at 8% will gain $64 in 1 year 3 months?

6. What principal at 6% will gain $61.20 in 2 years 6 months 18 days?

Let x = the principal, then $\frac{153}{1000}x$, or $\frac{153x}{1000} = \$61.20$

(a) Find the sum of the six answers.

* TO THE PUPIL.—Prove each answer obtained by finding the interest upon it for the given time at the given rate.

261. Algebra Applied to Some Problems in Interest.

Example.

What principal at 6 % will amount to $828.80 in 2 years?

Let x = the principal.

Then $x + \frac{12x}{100} = 828.80.$

Multiplying by 100 $100x + 12x = 82880.$

Uniting $112x = 82880$

$x = 740.$

Problems.

1. What principal at 6 % will amount to $368 in 2 years 6 months? *

2. What principal at 6 % will amount to $588.30 in 1 year 10 months?

3. What principal at 5 % will amount to $393.75 in 2 years 6 months?

Let x = the principal; then $\frac{12\frac{1}{2}}{100}x$, or $\frac{1}{8}x$, or $\frac{x}{8}$ = the interest;

and $x + \frac{x}{8}$ = the amount, $393.75.

4. What principal at 5 % will amount to $287.50 in three years?

5. What principal at 6% will amount to $458.80 in 2 years 5 months and 12 days?

Let x = the principal; then $\frac{147}{1000}x$, or $\frac{147x}{1000}$ = the interest,

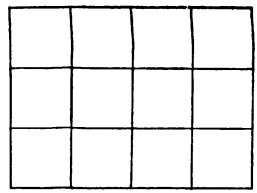
and $x + \frac{147x}{1000}$ = the amount, $458.80.

(a) Find the sum of the five answers.

* To the Pupil.—Prove each answer obtained by finding its amount for the given time at the given rate.

Geometry.

262. The Area of a Rectangle.



1. One side of every rectangle may be regarded as its base. The side perpendicular to its base is its altitude.

2. The number of square units in the row of square units next to the base of a rectangle, taken as many times as there are linear units in its altitude, equals the number of square units in its area. In the figure given, we have 4 sq. units × 3 = 12 sq. units.

NOTE 1.—In the above, it is assumed that the base and altitude are measured by the same linear unit, and that the sq. unit takes its name from the linear unit.

NOTE 2.—In the actual finding of the area of rectangles for practical purposes, the work is done mainly with abstract numbers and the proper interpretation is given to the result. There can be no serious objection to the rule for finding the area of rectangles, as given in the old books, provided the pupil is able to interpret it.

RULE.—To find the area of a rectangle, "*multiply its base by its altitude.*"

PROBLEMS.

1. Find the area of the surface of a cubical block whose edge is 9 inches in length.

2. Find the area in square yards, of a rectangular piece of ground that is 36 feet by 45 feet.

3. Find the area in acres, of a rectangular piece of land that is 92 rods by 16 rods.

4. Find the area in square rods of a piece of ground that is 99 feet by 66 feet.

5. The area of a field 30 rods square is how many times as great as the area of a field 10 rods square?

263. Miscellaneous Review.

1. Clarence Marshall wished to borrow some money at a bank. He was told by the president of the bank that they (the bank officials) were "discounting good 30-day paper" at 7 %. Mr. Marshall's name being regarded as "good," he drew his note upon one of the forms in use at the bank, for $1000 payable in thirty days without interest. On the presentation of this note to the cashier, how much money should he receive?

2. If Mr. Marshall's note described in problem 1 was dated April 10, 1898, (a) when must it be paid? (b) How much money will he pay when he "takes up" the note? (c) Does he pay for the use of the money borrowed, at the rate of exactly 7 % per annum?

3. If a bank is discounting at 7 %, how much should be given for a note of $200 due in two months from the time it is discounted and bearing interest at the rate of 6 % per annum from the date of discounting?

4. Find the value at the time of settlement of the following note:

Date of note, Apr. 1, 1896.
Face of note, $300.
Rate of interest, six per cent.
Payment, Aug. 1, 1896, $75.
Payment, Apr. 1, 1897, $80.
Settled, Aug. 1, 1898.

5. What principal at 6 % will gain $6 in 1 year 4 months?

6. What principal at 6 % will amount to $81 in 1 year 4 months?

7. Find the area in square feet, of a walk 4 feet wide around a rectangular flower bed that is 40 feet long and 12 feet wide.

STOCKS AND BONDS.

264. Some kinds of business require so much capital that many persons combine to provide the necessary money. Such a combination of men organized under the laws of a State, the capital being divided into shares, is known as **a corporation,** or **stock-company.** Those who own the shares are called **stock-holders.** The stock-holders elect from their own number, certain men to manage the business. These managers are called **directors.**

265. The **nominal value** of a share is its face value; that is, the sum named on its face. Large corporations, usually, though not always, divide their capital into $100 shares.

If the business is prosperous, shares may sell on the market for more than their nominal value. The stock is then said to be "*above par*," or "*at a premium.*"

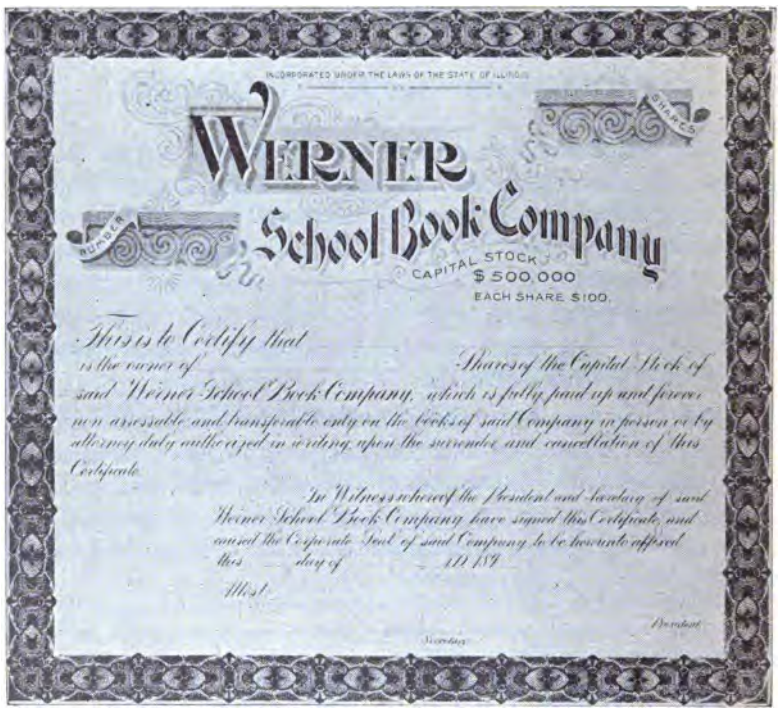
If the business does not prosper, the shares may sell on the market for less than their nominal value. The stock is then said to be "*below par*," or "*at a discount.*"

266. If the business is profitable, a part of all of the earnings is periodically divided among the stock-holders. The sum divided is called **a dividend.**

Dividends are always reckoned on the nominal or par value of the stock. If a corporation declares a 2 % dividend, it pays to each stock-holder a sum equal to 2 % of the nominal value of the stock which he owns.

267. The kinds of business which are usually conducted by corporations, are: The mining of coal, silver, gold, etc.; the operation of gas works, railroads, large manufacturing establishments of all kinds, creameries, etc.

Certificate of Stock.



1. Examine the above certificate. What part of the entire stock of the Werner School Book Company would the owner of one hundred shares have?

2. A 3 % dividend would require how much money from the treasury of the Company? How much money should the owner of one hundred shares receive?

3. If the owner of five hundred shares sold his stock at a premium of 8 % how much should he receive for it?

4. If a 4 % dividend is declared how much money should the owner of seven hundred and fifty shares of stock receive as his share of the dividend?

Stocks.

268. History of a Stock Company.

The farmers of a certain community agreed to combine in the building and management of a creamery. It was determined that a capital of $5000 was necessary. This was divided into 50 shares of $100 each. Men then came forward and contributed as follows.

A took 5 shares and paid in $500.
B took 3 shares and paid in $300.
C took 6 shares and paid in $600.
D took 4 shares and paid in $400.
E took 7 shares and paid in $700.
F took 2 shares and paid in $200.
G took 10 shares and paid in $1000.
H, I, J, K, L, M, N, O, P, Q, R, S, & T, took 1 share each and paid in $100 each.

(a) At the end of the first year the directors declared an 8 % dividend. How much did A receive? B? C? K?

(b) What was the entire amount of the money divided among the stock-holders at the end of the first year?

(c) Soon after this dividend was paid, A sold his stock to X at a premium of 10 %. How much did A receive for his stock?

(d) At the end of the second year the profits were found to be comparatively small, and the directors could pay only a 3% dividend. How much did X receive? B? C? K?

(e) What was the entire amount of the money divided among the stock-holders at the end of the second year?

(f) Soon after this dividend was paid, B, C, D, E, and F, sold their stock to Y at a discount of 10 %. How much did this stock cost Y?

(g) At the end of the third year, the profits were so small that no dividend was declared. The stock-holders became

History of a Stock Company, Continued.

disheartened and many of them offered to sell their stock at a large discount. Z appeared in the market and bought at "50¢ on the dollar" all the stock of the company except that owned by X and Y. How much did this stock cost Z?

(h) At the end of the fourth year, the directors, X, Y, and Z, declared a 10 % dividend. How much money was divided and how much did each receive?

(i) Before the close of the fifth year, the property burned and the lot upon which it stood was sold. After the insurance money had been received, the book accounts collected, and all debts paid, there remained in the treasury of the company $4350. How much of this money should each stock-holder, X, Y, and Z, receive?

(j) Did this creamery enterprise prove a good investment for X? for Y? for Z? for A? for B? for M?

Miscellaneous Problems.

1. The directors of a company whose capital is $50000 determined to distribute among the stock-holders $2500 of profits. (a) A dividend of what per cent shall be declared? (b) How much will a man receive who owns 15 100-dollar shares?

2. A company whose capital is $75000 pays a dividend of 3 %. (a) How much money is divided among the stock-holders? (b) How much does a man receive who owns 6 100-dollar shares?

3. Mr. Steele owns 20 shares ($100) in the C., B. & Q. R. R. He receives as his part of a certain dividend $110. What is the per cent of the dividend?

Note.—Stocks are sometimes bought and sold on commission. In such cases the commission is usually reckoned upon the par value as a base.

Bonds.

269. A **bond** is a very formal promissory note given by a government or other corporate body, as a railway or a gas company, for money borrowed. Bonds usually have attached to them small certificates called **coupons.** These are really little notes for the interest that will be due at different times. Thus, a 10-year bond for $1000 with interest at 6 % payable semi-annually will have 20 coupons attached, each calling for $30 of interest.

270. Money invested in bonds yields a specified income; but the income from money invested in stocks depends upon the profits of the company.

271. Bonds, like stocks, are sometimes sold for more than their face value. They are then said to be "*above par*" or "*at a premium.*" Like stocks, too, they are sometimes "*below par*," or "*at a discount.*"

1. What is the semi-annual interest on two 1000-dollar U. S. 5 % bonds?

2. What sum should be named on each coupon of a 1000-dollar city bond if the interest is payable annually at the rate of 7 %?

3. To raise the money to build a court-house a certain county issued $50000 worth of 6 % ten-year bonds. These sold upon the market at 2 % premium.* (a) How much money was received for the bonds? (b) How much did A pay, who bought three 1000-dollar bonds? (c) If the interest was payable semi-annually, how much should A receive each 6 months, on this investment?

4. Has the county or city in which you live any "bonded indebtedness"? If so, how much, and what is the rate of interest? Did the bonds sell at a premium, at par, or at a discount?

*Premium, discount, interest, and commission are all reckoned on the par value of the bond.

City Bond with Coupons Attached.

NOTE.—Bonds are made in great variety both as to form and content; but in all, indebtedness is acknowledged, and the amount, rate of interest, and time of payment for both principal and interest, named. The above is a very short and concise form of Bond (much reduced in size) and is an exact copy of one prepared for actual use.

1. Examine the above Bond. If the time it is to run is five years, how many coupons should be attached?

2. If the Bond is dated Jan. 1, 1898, what date should be written in each coupon?

3. If the face of the Bond is $100 and the rate 5 %, what sum should be written in each coupon?

4. If the rate is 5 % per annum, what is the half-yearly interest on a $10,000 bond?

Algebra.

272. Algebra Applied to Some Problems in Interest.

Example.

At what rate per cent will \$500 gain \$55 in 2 yrs?*

Let x = the rate,

then $\frac{2x}{100}$ of 500, or $\frac{1000x}{100}$, or $10x$ = the interest,

and $10x = 55$

Dividing $x = 5\frac{1}{2}$

Problems.

1. At what rate per cent will \$450 gain \$72 in 2 years?†
2. At what rate per cent will \$320 gain \$48 in 3 years?
3. At what rate per cent will \$560 gain \$84 in 2 years 6 months?
4. At what rate per cent will \$600 gain \$75 in 2 years 6 months?
5. At what rate per cent will \$600 gain \$114 in 2 years 4 months 15 days?

2 yr. 4 mo. 15 da. = $2\frac{3}{8}$ years.

Let x = the rate. Then $\frac{2\frac{3}{8}x}{100}$ of 600 = 114.

Note.—Problem 5 may be solved arithmetically by finding the interest of \$600 for 2 yr. 4 mo. 15 da. at 6 %. Divide this interest by 6 (to find the interest at 1 %) and find how many times the quotient is contained in \$114.

*The arithmetical solution of this problem is as follows: The interest of \$500 for 2 years at 1 % is $\frac{2}{100}$ of \$500. $\frac{2}{100}$ of \$500 = \$10. To gain \$55 in 2 years, \$500 must be loaned at as many per cent as \$10 is contained times in \$55. It is contained $5\frac{1}{2}$ times; so \$500 must be loaned at $5\frac{1}{2}$ % to gain \$55 in 2 years. *Observe* that by this method we *divide the given interest by the interest of the principal for the given time at one per cent.*

†To the Pupil.—Prove each answer by finding the interest on the given principal for the given time at the rate obtained.

Algebra.

273. Algebra Applied to some Problems in Interest.

Example.

In how long a time will $650 gain $97.50 at 6 %?

Let x = the number of years.

then $\frac{6x}{100}$ of 650 = 97.50

Simplifying $39x = 97.50$

Dividing $x = 2.5$*

Problems.

1. In how long a time will $400 gain $30 at 5 %?
2. In how long a time will $600 gain $96 at 6 %?
3. In how long a time will $800 gain $68 at 6 %?
4. In how long a time will $500 gain $56 at 6 %?
5. In how long a time will $400 gain $29 at 6 %?

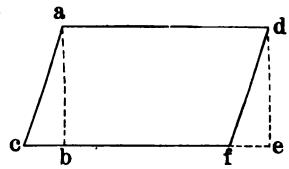
Review Problems.

6. What principal at 8 % will gain $124.80 in 3 years? (See page 167.)
7. What principal at 7 % will amount to $410.40 in 2 years? (See page 168.)
8. At what rate per cent will $900 gain $72 in 2 years? (See page 177.)
9. In how long a time will $1000 gain $160 at 6 per cent? (See above.)

To the Pupil.—Prove each answer by finding the interest on the given principal at the given rate for the time obtained.

* The arithmetical solution of this problem is as follows: The interest of $650 for one year at 6 % is $39. As many years will be required to gain $97.50 as $39.00 is contained times in $97.50. It is contained 2½ times; so in 2½ years $650 will gain $97.50. Observe that by either method we *divide the given interest by the interest of the principal for 1 year at the given rate.*

274. THE AREA OF A RHOMBOID.*



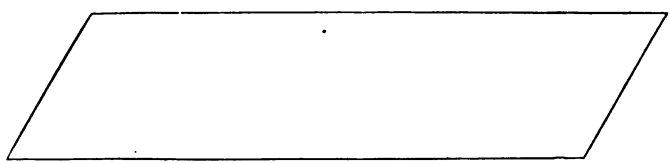
1. One side of a rhomboid may be regarded as its *base*. The perpendicular distance from the base to the opposite side, is its *altitude*.

In the figure here given cf is the base and ab the altitude.

2. Convince yourself by measurements and by paper-cutting that from every rhomboid there may be cut a triangle, (*abc*), which when placed upon the opposite side, (*def*), converts the rhomboid into a rectangle (*adeb*).

Observe that the base of the rectangle is equal to the base of the rhomboid, and the altitude of the rectangle equal to the altitude of the rhomboid.

3. A rhomboid is equivalent to a rectangle having the same base and altitude. Hence, to find the area of a rhomboid, *find the area of a rectangle whose base and altitude are the same as the base and altitude of the rhomboid.* Or, as the rule is given in the older books,—"*Multiply the base by the altitude.*"



PROBLEM.—If the above figure represents a piece of land, and is drawn on a scale of $\frac{1}{8}$ inch to the rod, how many acres of land?

* The statements upon this page apply to the rhombus as well as to the rhomboid.

275. Miscellaneous Review.

1. Mr. Watson purchased 15 shares of C., B. & Q. R. R. stock at 12 % discount. (a) How much did he pay for the stock? (b) When a 3 % dividend is declared and paid, how much does he receive?*

2. James Cooper bought 12 shares of stock in the Sugar Grove Creamery at 8 % below par, and a few days after sold the stock at 5 % above par. How much more did he receive for the stock than he gave for it?

3. A certain city borrowed a large sum of money and issued therefor 10-year 5 % bonds with the interest payable semi-annually. (a) How many coupons were attached to each bond? (b) On a $1000 bond, each coupon should call for how much money?

4. Sometimes such bonds as those described in problem 3, are offered for sale to the highest bidder, in "blocks" of $10000, $20000, or $50000. If a $20000 "block" is "bid off" at 2¼% premium, how much should the city receive for the "block"?

5. What must be the nominal value of 5 % bonds that will yield to their owner an annual income of $750?

Let x = the nominal value; then $\frac{5x}{100} = \$750$.

6. What must be the nominal value of 4 % bonds that will yield to their owner an annual income of $720?

7. A owns $6000 of 5 % bonds; B owns $8000 of 4½ % bonds. How much greater is the annual income from B's bonds than from A's?

8. Find the area of a piece of land in the form of a rhomboid, whose base is 32 rods and whose altitude is 15 rods.

9. Find the area of a piece of land in the form of a rectangle, whose base is 32 rods and whose altitude is 15 rods.

* The par value of each share of stock mentioned on this page is $100.

RATIO.

276. Ratio is relation by quotient. The two numbers (magnitudes) of which the ratio is to be found are called the **terms of the ratio.** The first term is called the **antecedent** and the second term the **consequent.** The ratio is the quotient of the antecedent divided by the consequent.

The usual sign of ratio is the colon. It indicates that the ratio of the two numbers between which it stands is to be found, the number preceding the colon being the antecedent and the number following it, the consequent. The expression, 12 : 4 = 3, is read, the ratio of 12 to 4 is 3.

EXERCISE.

Read and complete the following:

1.	12 : 4 =	4:12 =	12 : 2 =
2.	18 : 9 =	9:18 =	18 : 6 =
3.	15 : 5 =	5:15 =	15:10 =
4.	40:10 =	10:40 =	40:20 =

NOTE.—It will be observed that the sign of ratio is the sign of division (÷) with the line omitted.

277. *Every integral number is a ratio.* The number 4 is the ratio of a magnitude 4 (inches, ounces, bushels,) to the measuring unit 1 (inch, ounce, bushel). The number 7 is the ratio of 7 yards to 1 yard; of 7 dollars to 1 dollar, or of 7 seconds to 1 second, etc.

NOTE.—The ratio aspect of numbers is not the aspect most frequently uppermost in consciousness: neither ought it to be. But the pupil should now see that number *is* ratio; that while it implies aggregation and often stands in consciousness for magnitude, its essence is relation—ratio.

Ratio.

278. *Every fractional number is a ratio.* The fraction $\frac{3}{4}$ is the ratio of the magnitude 3 to the magnitude 4.

So $\frac{12}{4}$, (3), is the ratio of 12 to 4. *Observe* that in every case the terms of a ratio may be written as the terms of a fraction; the antecedent becoming the numerator and the consequent the denominator of the fraction. *The fraction itself is the ratio.*

Exercise I.

Make the terms of the ratio the terms of a fraction; then reduce the fraction to its simplest form.

1. The ratio of 20 to 6 is $\frac{20}{6} = \frac{10}{3} = 3\frac{1}{3}$.
2. The ratio of 6 to 20 is $\frac{6}{20} = \frac{3}{10}$.
3. The ratio of 7 to 5 is —; of 5 to 7, —.
4. The ratio of 12 to 1 is —; of 1 to 12, —.

Exercise II.

1. $\frac{5}{7}$ is the ratio of 5 to 7; of 10 to 14; of 15 to 21, etc.
2. $\frac{3}{4}$ is the ratio of — to —; of — to —; of — to —, etc.
3. $\frac{6}{5}$ is the ratio of — to —; of — to —; of — to —, etc.
4. 8 is the ratio of 8 to 1; of — to —; of — to —, etc.

Exercise III.

Make the necessary reduction and find the ratio: *

1. Of 2 feet to 8 inches.
2. Of 3 yards to 6 inches.
3. Of 6 rods to 3 yards.
4. Of 2 rods 5 yards, to 1 yard 1 foot.
5. Of 3 miles 40 rods to 1 mile 80 rods.

* The comparison of two magnitudes involves their measurement by the same standard. To compare feet with inches, the inches may be changed to feet or the feet to inches, or both may be changed to yards.

Ratio.

279. Not only is number itself ratio, but a large part of the work in arithmetic is merely the changing of the form of the expression of ratios.

Exercise IV.

(Reducing fractions to their lowest terms.)

1. Express the ratio of 30 to 40 in its simplest form.
2. Express the ratio of 560 to 720 in its simplest form.
3. Express the ratio of 425 to 875 in its simplest form.
4. Express the ratio of 5 min. to 2 hours in its simplest form.
5. Express the ratio of 1 lb. 4 oz. to 5 lb. 8 oz. in its simplest form.

Exercise V.

(Reducing improper fractions to integers.)

1. Express the ratio of 400 to 50 in its simplest form.
2. Express the ratio of 375 to 25 in its simplest form.
3. Express the ratio of 256 to 16 in its simplest form.
4. Express the ratio of 3 hours 20 minutes to 50 minutes in its simplest form.
5. Express the ratio of 8 lb. 12 oz. to 1 lb. 4 oz. in its simplest form.

Exercise VI.

(Reducing complex fractions to simple fractions.)

1. Express the ratio of $\frac{1}{2}$ to $\frac{3}{4}$ in its simplest form.
2. Express the ratio of $\frac{3}{5}$ to $\frac{3}{8}$ in its simplest form.
3. Express the ratio of $2\frac{1}{3}$ to $8\frac{1}{2}$ in its simplest form.
4. Express the ratio of $\frac{3}{4}$ of an inch to 1 foot in its simplest form.

Note.—Observe that the denominator in fractions corresponds to the consequent in ratio.

Ratio.

EXERCISE VII.

(Changing common fractions to decimals.)

1. Express the ratio of 3 to 4 ($\frac{3}{4}$), in hundredths.
2. Express the ratio of 20 to 50, in tenths.
3. Express the ratio of 30 to 80, in thousandths.
4. Express the ratio of 50 sq. rd. to 1 acre 40 rd., in hundredths.
5. Express the ratio of 4 ft. 6 in. to 5 yd., in tenths.

EXERCISE VIII.

(Finding what per cent one number is of another.)

1. Express the ratio of 15 to 20, in hundredths.
2. Express the ratio of 14 to 200, in hundredths.
3. Express the ratio of 17 to 25, in hundredths.
4. Express the ratio of 16 to $33\frac{1}{3}$, in hundredths.
5. Express the ratio of 27 to 500, in hundredths.

EXERCISE IX.

(Changing "per cent" to a common fraction in its lowest terms, or to a whole or mixed number.)

1. A's money equals 40 % of B's money. (a) Express the ratio of A's money to B's money in the form of a fraction in its lowest terms. (b) Express the ratio of B's money to A's money in its simplest form.

2. One number is 50 % more than another number. (a) Express the ratio of the smaller to the larger number in the form of a fraction in its lowest terms. (b) Express the ratio of the larger to the smaller number in its simplest form.

NOTE.—Observe that the base in percentage corresponds to the consequent in ratio.

Ratio.

EXERCISE X.

(The specific gravity of a liquid or solid is the ratio of its weight to the weight of the same bulk of water. See Book II., pp. 214 and 224.)

1. A cubic foot of water weighs 62½ lbs. A cubic foot of cork weighs 15 lbs. What is the ratio of the weight of the cork to the weight of the water? Express the ratio in hundredths. What is the specific gravity of cork?

2. A certain piece of limestone weighs 37 ounces. Water equal in bulk to the piece of limestone weighs 15 ounces. What is the ratio of the weight of the limestone to the weight of the water? What is the specific gravity of the limestone?

3. A certain bottle holds 10 ounces of water or 9½ ounces of oil. What is the ratio of the weight of the oil to the weight of the water? Express the ratio in hundredths. What is the specific gravity of the oil?

NOTE — *Observe* that the weight of water in specific gravity problems, corresponds to the consequent in ratio problems.

280. MISCELLANEOUS QUESTIONS.

1. What is the ratio of a unit of the first integral order to a unit of the first decimal order?

2. What is the ratio of a unit of any order to a unit of the next lower order?

3. What ratio corresponds to 6 per cent?

4. What is the ratio of a dollar to a dime? Of a dime to a cent? Of a cent to a mill?

5. What is the ratio of a meter to a decimeter? Of a decimeter to a centimeter?*

6. What is the ratio of a rod to a yard? Of a yard to a foot? Of a foot to an inch?

*See Book II., p. 154.

Ratio.

281. Some Old Problems in New Forms.*

1. What is the ratio of the area of a 2-inch square to the area of a 6-in. square? * Of a 6-in. square to a 2-in. square?

2. What is the ratio of the perimeter of a 2-in. square to the perimeter of a 6-in. square? Of the perimeter of a 6-in. square to the perimeter of a 2-in. square?

3. What is the ratio of the area of a 3-cm. square to the area of a 6-cm. square? Of a 6-cm. square to a 3-cm. square? †

4. What is the ratio of the perimeter of a 3-cm. square to the perimeter of a 6-cm. square? Of the perimeter of a 6-cm. square to the perimeter of a 3-cm. square?

5. What is the ratio of the solid content of a 2-inch cube to the solid content of a 6-in. cube? Of a 6-in. cube to a 2-in. cube?

6. What is the ratio of the surface of a 2-in. cube to the surface of a 6-in. cube? Of the surface of a 6-in cube to the surface of a 3-in. cube?

7. What is the ratio of the solid content of a 3-cm. cube to the solid content of a 6-cm. cube? Of a 6-cm. cube to a 3-cm. cube? ‡

8. What is the ratio of the surface of a 3-cm. cube to the surface of a 6-cm. cube? Of the surface of a 6-cm. cube to the surface of a 3-cm. cube?

9. What is the ratio of a square inch to a square foot? Of a cubic inch to a cubic foot?

* If pupils image the magnitudes compared, they will find no difficulty in the solution of these problems.

† See Book II., pp. 154 and 164. ‡ See Book II., p. 174.

Algebra.

282. Algebra Applied to some Problems in Ratio.

Example I.

The consequent is c; the ratio is r. What is the antecedent?

Let x = the antecedent.

Then $\frac{x}{c} = r$

and $x = cr$ the antecedent.

From the above learn that the antecedent is always equal to the product of the consequent and the ratio.

1. Consequent 75; ratio 11. Antecedent ?
2. Consequent 92; ratio $\frac{3}{4}$. Antecedent ?
3. Consequent .56; ratio $\frac{1}{8}$. Antecedent ?

Example II.

The antecedent is a; the ratio is r. What is the consequent?

Let x = the consequent.

Then $\frac{a}{x} = r$

and $a = rx$, or $rx = a$

dividing by r $x = \frac{a}{r}$ the consequent.

From the above learn that the consequent is always equal to the quotient of the antecedent divided by the ratio.

1. Antecedent 75; ratio 5. Consequent ?
2. Antecedent 96; ratio $\frac{3}{4}$. Consequent ?
3. Antecedent $\frac{2}{5}$; ratio $\frac{3}{8}$. Consequent ?
4. Antecedent .25; ratio $\frac{3}{5}$. Consequent ?

283. To Find Two Numbers when their Sum and Ratio are Given.

Example.

The sum of two numbers is 36 and their ratio is 3. What are the numbers?

Let x = the smaller number.
Then $3x$ = the larger number,
and $x + 3x = 36$
$4x = 36$
$x = 9$, the smaller number.
$3x = 27$, the larger number.

Problems.

1. The sum of two numbers is 196, and their ratio is 3. What are the numbers?

2. The sum of two numbers is 294, and their ratio is $2\frac{1}{2}$. What are the numbers?

3. The sum of two decimals is .42, and their ratio is $2\frac{1}{2}$. What are the decimals?

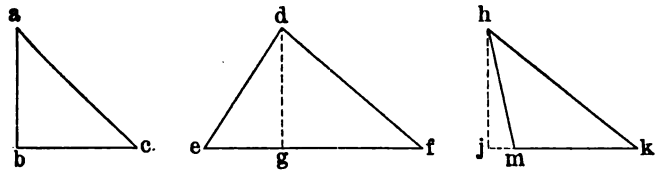
4. The sum of two numbers is s, and the ratio of the larger to the smaller is r. What are the numbers?

Let x = the smaller number.
Then rx = the larger number,
and $rx + x = s$
or $(r+1)x = s$
Dividing $x = \frac{s}{r+1}$ *

* Observe that any number you please may be put in the place of s, and any number greater than 1 in the place of r; so that when the sum of two numbers and the ratio of the larger to the smaller are given, the smaller number may be found by dividing the sum by the ratio plus 1.

Geometry.

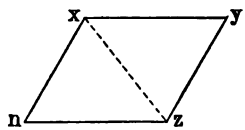
284. The Area of a Triangle.



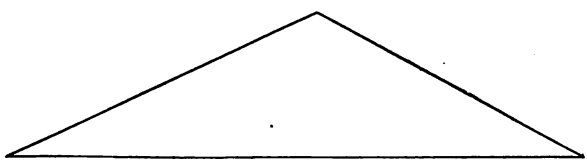
1. One side of a triangle may be regarded as its base. *The perpendicular* distance from its base (or from its base extended) to the opposite angle, is its altitude.

2. What is the altitude of the first of the above triangles? Of the second? Of the third?

3. Convince yourself by measurment and by paper cutting that every triangle is one half of a parallelogram having the same base and the same altitude as the triangle.



4. To find the area of a triangle, *Find the area of the parallelogram having the same base and altitude, and take one half of the result.* Or, as the rule is given in the older books,—"*Take one half of the product of the base and altitude.*"



Problem.—If the above figure represents a piece of land and is drawn on a scale of $\frac{1}{8}$ inch to the rod, what part of an acre of land does it represent?

285. Miscellaneous Review.

1. The specific gravity of granite is 2.7.* How much does a cubic foot of granite weigh?

2. A certain vessel is exactly large enough to contain 1000 grains of water. It will contain only 700 grains of petroleum. What is the specific gravity of the petroleum? †

3. The specific gravity of gold is 19.3. How much does a cubic foot of gold weigh?

4. A cubic foot of sulphur weighs 125 lbs. What is the specific gravity of sulphur?

5. A cubic foot of steel weighs 487.5 lbs. What is the specific gravity of steel?

6. What is the ratio of .01 to .001? Of .1 to .01? Of 1 to .1?

7. What is the ratio of 1 bu. to 1 pk.? Of 1 pk. to 1 qt.?

8. What is the ratio of $37\frac{1}{2}$ to 15? Of 15 to $37\frac{1}{2}$?

9. What is the area of a rhomboid whose base is 16 inches and whose altitude is 16 inches?

10. Is the rhomboid described in problem 9 equilateral?

11. The ratio of the perimeter of one square to the perimeter of another square is 4. What is the ratio of the areas of the two squares?

12. Draw three triangles, the base of each being 4 inches and the altitude of each being 2 inches. Make one of them a right-triangle; another, an isosceles triangle, and the third having angles unlike either of the other two. What can you say of the area of the right-triangle as compared with each of the others?

* See page 185, Exercise 10. † This means what is the ratio of the weight of the petroleum to the weight of the same bulk of water?

PROPORTION.

286. The terms of a ratio are together called **a couplet.** Two couplets whose ratios are equal are called **a proportion.**

The two couplets of a proportion are often written thus: $6 : 18 = 10 : 30$, and should be read, the ratio of 6 to 18 equals the ratio of 10 to 30.

Couplets are sometimes written thus: $20 : 4 :: 50 : 10$, and read, 20 is to 4 as 50 is to 10.*

287. To Find a Missing Term in a Proportion.

Example I.

$$36 : 12 :: x : 25.$$

The ratio of the first couplet is 3; that is, the antecedent is 3 times the consequent. Since the ratios of the couplets are equal, the ratio of the second couplet must be 3, and its antecedent must be 3 times its consequent. Three times 25 = 75, the missing term.

Problems.

Find the missing term.

1. $90 : 45 :: x : 180$
2. $48 : 12 :: x : 150$
3. $75 : 30 :: x : 140$
4. $20 : 60 = x : 225$
5. $30 : 50 = x : 175$
6. $90 : 20 = x : 140$

* The ratio sign (:) may be regarded as the sign of division (÷) with the horizontal line omitted, and the proportion sign (::) the sign of equality (=) with an erasure through its center, thus: (= =).

Proportion.

EXAMPLE II. $36 : 12 = 48 : x$.

Since the ratio of the first couplet is 3, the ratio of the second couplet must be 3, and x must equal 1 third of 48. 1 third of 48 is 16.

PROBLEMS.

Find the missing term.

1. $84 : 21 = 172 : x$
2. $96 : 16 :: 45 : x$
3. $75 : 30 = 125 : x$ *
4. $20 : 60 :: 120 : x$
5. $25 : 35 = 45 : x$
6. $50 : 25 :: 14 : x$

EXAMPLE III. $36 : x = 45 : 15$.

The ratio of each couplet is 3; so each consequent must be 1 third its antecedent, and x, 1 third of 36, or 12.

PROBLEMS.

Find the missing term.

1. $54 : x = 90 : 30$
2. $75 : x :: 125 : 25$
3. $50 : x = 40 : 16$
4. $18 : x :: 65 : 195$
5. $50 : x = 12 : 18$
6. $35 : x :: 21 : 3$

EXAMPLE IV. $x : 12 = 100 : 25$.

The ratio of each couplet is 4; so each antecedent must be 4 times its consequent, and x, 4 times 12, or 48.

PROBLEMS.

Find the missing term.

1. $x : 16 :: 51 : 17$
2. $x : 22 = 76 : 19$
3. $x : 11 :: 24 : 3$
4. $x : 96 = 23 : 92$
5. $x : 40 :: 36 : 48$
6. $x : 27 = 42 : 14$

* Since the ratio is $2\frac{1}{2}$ ($\frac{5}{2}$) the consequent must be $\frac{2}{5}$ of the antecedent.

288. PRACTICAL PROBLEMS.

1. If 75 yd. of cloth cost \$115.25, how much will 15 yd. cost at the same rate?

$$75 \text{ yd.} : 15 \text{ yd.} = \$115.25 : x$$

2. If $2\frac{1}{2}$ acres of land cost \$76.20, how much will 15 acres cost at the same rate?

3. If 7 tons of coal can be bought for \$26, how many tons can be bought for \$39?

$$7 \text{ tons} : x \text{ tons} : : \$26 : \$39.$$

4. If 36 lb. coffee can be bought for \$7, how many pounds can be bought for $\$17\frac{1}{2}$?

5. If sugar sells at the rate of 18 lb. for \$1, how much should 63 lb. of sugar cost?

$$18 \text{ lb.} : 63 \text{ lb.} = \$1 : x \text{ dollars.}$$

6. If a post 6 ft. high casts a shadow 4 feet long, how high is that telegraph pole which at the same time and place casts a shadow 20 feet long?

7. If a post 5 feet high casts a shadow 8 feet long, how high is that steeple which casts a shadow 152 feet long?

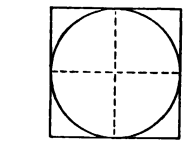
8. If a train moves 50 miles in 1 hr. 20 min., at the same rate how far would it move in 2 hours?

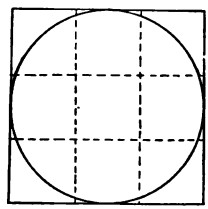
9. If a boy riding a bicycle at a uniform rate goes 12 miles in 1 hr. 15 min., how far does he travel in 25 minutes?

TO THE TEACHER —After the pupil has solved the above problems by making use of the fact of the equality of the ratios, he should solve them by an analysis somewhat as follows: Prob. 1. Since 75 yd. cost \$115.25, 1 yd. costs $\frac{1}{75}$ of \$115.25; but 15 yd. cost 15 times as much as 1 yd., so 15 yd. cost 15 times $\frac{1}{75}$ of \$115.25.

289. Magnitudes Which Are Proportional to the Squares of Other Magnitudes.

The areas of two squares are to each other as the squares of their lengths.





The areas of two circles are to each other as the squares of their diameters.

Observe that the ratio of the areas of the above squares is $\frac{4}{8}$ (or $\frac{9}{4}$). But the area of each circle is .78 + of its circumscribed square; so the ratio of the areas of the circles is $\frac{4}{8}$ (or $\frac{9}{4}$). See Book II, p. 256.

Problems.

1. The diameters of two circles are as 3 to 4. Compare their areas.

2. The area of a 6-inch circle is how many times as great as the area of a 3-inch circle?

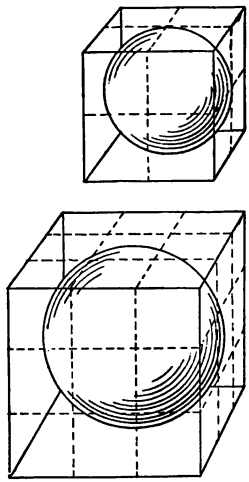
3. If a 4-inch circle of brass plate weighs 3 ounces, how much will a 6-inch circle weigh, the thickness being the same in each case?

4. If a piece of rolled dough 1 foot in diameter is enough for 17 cookies, how many cookies can be made from a piece 2 feet in diameter, the thickness of the dough and the size of the cookies being the same in each case?

5. If a piece of wire $\frac{1}{8}$ of an inch in diameter will sustain a weight of 1000 lbs., how many pounds will a wire $\frac{1}{4}$ of an inch in diameter sustain?

Proportion.

290. Magnitudes Which Are Proportional to the Cubes of Other Magnitudes.



The solid contents of two cubes are to each other as the cubes of their lengths.

The solid contents of two spheres are to each other as the cubes of their diameters.

Observe that the ratio of the solid contents of the above cubes is $\frac{8}{27}$ (or $\frac{27}{8}$). But the solid content of each sphere is .52+ of its circumscribed cube; so the ratio of the solid contents of the spheres is $\frac{8}{27}$ (or $\frac{27}{8}$). See Book II, page 266.

Problems.

1. The diameters of two spheres are as 3 to 4. Compare their solid contents.

2. The solid content of a 6-inch sphere is how many times as great as the solid content of a 3-inch sphere?

3. If a 4-inch sphere of brass weighs 10 lbs., how many pounds will a 6-inch sphere of brass weigh?

4. If a sphere of dough 1 foot in diameter is enough for 20 loaves of bread, how many loaves can be made from a sphere of dough 2 feet in diameter?

5. If the half of a solid 8-inch globe weighs 4 lbs., how much will the half of a solid 5-inch globe weigh, the material being of the same quality?

291. Magnitudes Which Are Inversely Proportional to Other Magnitudes or to the Squares of Other Magnitudes.

Example.

If 5 men do a piece of work in 16 days, how long will it take 8 men to do a similar piece of work?

Operation and Explanation.

It is evident that the time required will be inversely proportional to the number of men employed; that is, if twice as many men are employed, not twice as much, but $\frac{1}{2}$ as much time will be required. Hence the proportion is not $5 : 8 = 16 : x$, but, $5 : 8 = x : 16$; hence $5 : 8 = 10 : 16$.

The interpretation of the above equation is, if 5 men can do a piece of work in 16 days, 8 men can do it in 10 days.

Problems.

1. If 4 men can do a piece of work in 20 days, how long will it take 5 men to do a similar piece of work?

2. If 8 men can do a piece of work in 12 days, how long will it take 3 men to do a similar piece of work?

It can be shown that the intensity of light upon an object diminishes as the square of the distance between the luminous body and the illuminated object increases; that is, if the distance be twice as great in one case as in another, the intensity is not twice as great, not $\frac{1}{2}$ as great, but $\frac{1}{4}$ as great; if the distances are as 2 to 3 the intensities are, not as 2 to 3, not as 3 to 2, but as 9 to 4. The intensity at 2 feet is $\frac{9}{4}$ as great as at 3 feet.

3. Object A is 15 feet from an incandescent electric light. Object B is 20 feet from the same light. Object C is 30 feet from the same light. (a) How does the intensity of the light at B compare with the intensity at A? (b) How does the intensity at C compare with the intensity at A?

Algebra.

292. To Find the Missing Term of a Proportion Without Finding the Ratio.

The first and fourth terms of a proportion are called the **extremes,** and the second and third terms, the **means;** thus, in the proportion $12 : 6 = 8 : 4$, 12 and 4 are the extremes, and 6 and 8 are the means.

Observe that in the following proportions the product of the means equals the product of the extremes:

$6 : 3 = 8 : 4$; then $6 \times 4 = 3 \times 8$
$2 : 5 = 4 : 10$; then $2 \times 10 = 5 \times 4$
$\frac{1}{2} : \frac{1}{4} = 4 : 2$; then $\frac{1}{2} \times 2 = \frac{1}{4} \times 4$

Let $a : b = c : d$, stand for any proportion.

Then $$\frac{a}{b} = \frac{c}{d}$$

Clearing of fractions, $ad = bc$

But a and d are the extremes, and b and c, the means; hence, in any proportion, in which abstract numbers are employed *the product of the means equals the product of the extremes.*

Example I.

$30 : 20 = 18 : x$
$30x$ = the product of the extremes.
20×18 = the product of the means.
Then $30x = 20 \times 18$, or 360,
and $x = 12$

Example II.

$10 : 25 = x : 50$
Then $25x = 10 \times 50$, or 500
and $x = 20$

Algebra.

EXAMPLE III.

$40 : x = 25 : 5$

Then $25x = 40 \times 5$, or 200

and $x = 8$

EXAMPLE IV.

$x : 35 = 4 : 28$

Then $28x = 35 \times 4$, or 140

and $x = 5$

PROBLEMS.

Find the missing terms:

1. $24 : 72 = x : 69$
2. $45 : 12 = 75 : x$
3. $35 : x = 14 : 40$
4. $x : 70 = 3 : 21$
5. $55 : 25 = x : 10$
6. $\frac{2}{3} : 4 = x : 30$
7. $x : \frac{3}{4} = 40 : 6$
8. $\frac{1}{2} : \frac{2}{5} = x : 8$
9. $.6 : .8 = 15 : x$
10. $.25 : 5 = x : 40$

11. If 8 acres of land cost $360, how much will 15 acres cost at the same rate?

$$8 : 15 = 360 : x^*$$

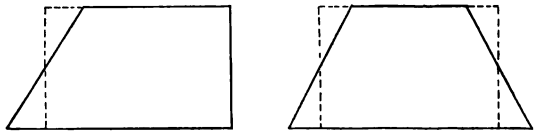
12. If 12 horses consume 3500 lb. of hay per month, how many pounds will 15 horses consume?

13. If 11 cows cost $280.50, how many cows can be bought for $433.50 at the same rate?

* Observe that in the solution of concrete problems by the method here given the numbers must be regarded as abstract. It would be absurd to talk or think of finding the product of 15 acres and 360 dollars and dividing this by 8 acres. It is true, however, that the ratio of 8 acres to 15 acres equals the ratio of 360 dollars to x dollars. It is also true that in the proportion $8 : 15 = 360 : x$, the product of the means is equal to the product of the extremes.

Geometry.

293. The Area of a Trapezoid.



1. Convince yourself by measurement and by paper cutting that from every trapezoid there may be cut a triangle (or triangles) which when properly adjusted to another part (or parts) of the trapezoid, will convert the trapezoid into a rectangle.

2. Convince yourself that the rectangle made from a trapezoid is not so long as the longer of the parallel sides of the trapezoid, and not so short as the shorter of the parallel sides of the trapezoid—that *its length is midway between the lengths of the two parallel sides of the trapezoid.*

Note.—*Observe* that the length of the rectangle thus formed may be found by adding half the difference of the parallel sides of the trapezoid to its shorter side, or by dividing the sum of its parallel sides by 2.

3. To find the area of a trapezoid, *find the area of the rectangle to which it is equivalent*, or, as the rule is usually given,—"*Multiply one half the sum of the parallel sides by the altitude.*"

Problems.

1. Find the area of a trapezoid whose parallel sides are 10 inches and 15 inches respectively, and whose altitude is 8 inches.

2. How many acres in a trapezoidal piece of land, the parallel sides being 28 rods and 36 rods respectively, and the breadth (altitude) 25 rods?

294. Miscellaneous Review.

1. If 3 men can build 72 feet of sidewalk in a day, how many feet can 4 men build?

2. If 3 men can do a piece of work in 12 hours, in how many hours can 4 men do an equal amount of work?

3. If a piece of land 8 rods square is worth $500, how much is a piece of land 16 rods square worth at the same rate?*

4. If a ball of yarn 3 inches in diameter is enough for a pair of stockings, how many pairs of stockings can be made from a ball 6 inches in diameter?†

5. If a grindstone 12 inches in diameter weighs 40 lb., how much will a grindstone 18 inches in diameter weigh, the thickness and quality of material being the same?

6. The opening in an 8-inch drain tile is how many times as large as the opening in a 2-inch drain tile?‡

7. Find the area of a rhomboidal piece of land whose length (base) is 64 rods, and whose width (altitude) is 15 rods.

8. Find the area of a trapezoidal piece of land, the length of the parallel sides being 44 rods and 52 rods respectively, and the width (altitude) being 18 rods.§

9. Find the area of a triangular piece of land whose base is 42 rods and whose altitude is 20 rods.

* Make diagrams of these pieces of land on a scale of 4 rods to the inch.

† Compare a 3-inch cube and a 6-inch cube. Remember that a 3-inch sphere is a little more than half of a 3-inch cube, and a 6-inch sphere a little more than one half of a 6-inch cube.

‡ Compare a 6-inch square with a 2-inch square. Remember that a 2-inch circle is about $\frac{3}{4}$ of a 2-inch square, and an 8-inch circle about $\frac{3}{4}$ of an 8-inch square.

§ Draw a diagram of the land on a scale of $\frac{1}{4}$ inch to the rod.

POWERS AND ROOTS.

295. A product obtained by using a number twice as a factor is called **the second power** or **the square** of the number; thus, 25, (5×5), is the second power, or the square of 5.

NOTE.—Twenty-five is called the *second* power of 5, because it may be obtained by using 5, *twice* as a factor. It is called the *square* of 5, because it is the number of square units in a square whose side is 5 linear units.

1. What is the second power of 2? 8? 3? 5?
2. What is the square of 4? 7? 1? 6? 9? 10?

$11^2 = ?$ $12^2 = ?$ $13^2 = ?$ $14^2 = ?$
$15^2 = ?$ $16^2 = ?$ $17^2 = ?$ $18^2 = ?$

(a) Find the sum of the eighteen squares.

296. The **square root** of a number is one of the two equal factors of the number.

The radical sign, $\surd$, (without a figure above it) indicates that the square root of the number following it, is to be taken; thus $\surd 64$, means the square root of 64.

1. What is the square root of 144? 81? 49?
2. What is the square root of 36? 25? 16?

$\surd 9 = ?$ $\surd 64 = ?$ $\surd 121 = ?$ $\surd 100 = ?$
$\surd 4 = ?$ $\surd 1 = ?$ $\surd 400 = ?$ $\surd 169 = ?$

(b) Find the sum of the fourteen results.

Powers and Roots.

297. Any number that can be resolved into two equal factors is a **perfect square.**

1. Tell which of the following are perfect squares and which are not:

9, 10, 12, 16, 18, 25, 32, 36.

NOTE.—It is a curious fact that no number, either integral or mixed, can be found which, when multiplied by itself, will give as a product 10, or 12, or 14, or any number that is not a *perfect square.*

2. Any integral number that is a perfect square is composed of an even number of like prime factors; that is, its prime factors are an even number of 2's, 3's, 5's, 7's, etc.

3. Tell which of the following are perfect squares?

144, $(2 \times 2 \times 2 \times 2 \times 3 \times 3)$; 250, $(2 \times 5 \times 5 \times 5)$; 225, $(5 \times 5 \times 3 \times 3)$.

RULE.—*To find the square root of an integral number, that is a perfect square, resolve the number into its prime factors and take half of them as factors of the root; that is, one half as many 2's, 3's, or 5's, etc., as there are 2's, 3's, or 5's, etc., in the factors of the number.*

4. Find the square root of 1225.

$1225 = 5 \times 5 \times 7 \times 7.$ $\quad \sqrt{1225} = 5 \times 7 = 35.$

5. Find the square root of 441; of 400.
6. Find the square root of 576; of 324.
7. Find the square root of 784; of 2025.
8. Find the square root of 625; of 3025.

(a) Find the sum of the last eight results.

Powers and Roots.

298. The Square of Common Fractions.

1. The square of $\frac{1}{2}$, $(\frac{1}{2} \times \frac{1}{2})$, is — —.

NOTE.—A square whose side is $\frac{1}{2}$ (of a linear unit) has an area of $\frac{1}{4}$ (of a square unit). Show this by diagram.

2. Answer the following and illustrate by diagram if necessary:

$(\frac{1}{3})^2 = ?$ $(\frac{1}{4})^2 = ?$ $(\frac{1}{5})^2 = ?$ $(\frac{1}{6})^2 = ?$

$(\frac{2}{3})^2 = ?$ $(\frac{3}{4})^2 = ?$ $(\frac{2}{5})^2 = ?$ $(\frac{5}{6})^2 = ?$

(a) Find the sum of the eight results.

3. A square of sheet brass whose edge is $\frac{5}{12}$ of a foot is what part of a square foot?

299. The Square Root of Common Fractions.

1. The square root of $\frac{9}{16}$ is — —.

NOTE 1.—A square whose area is $\frac{9}{16}$ (of a square unit) is $\frac{3}{4}$ (of a linear unit) in length. Show this by diagram.

NOTE 2.—Only those fractions are perfect squares, which, when in their lowest terms, have perfect squares for numerators and perfect squares for denominators.

2. What is the square root of $\frac{25}{36}$? Of $\frac{49}{36}$? Of $\frac{1}{4}$?

$\sqrt{\frac{144}{36}} = ?$ $\sqrt{\frac{169}{36}} = ?$ $\sqrt{\frac{81}{16}} = ?$ $\sqrt{\frac{121}{144}} = ?$

(b) Find the sum of the seven results.

3. The area of a square piece of sheet brass is $\frac{81}{144}$ of a square foot. What is the length of the side of the square?

4. How long is the side of a square of zinc the area of which is $\frac{25}{36}$ of a square yard?

5. What is the perimeter of a square piece of land whose area is $\frac{49}{36}$ of a square mile?

Powers and Roots.

300. The Square of Decimals.

1. The square of .5 is — —.

Note.—A square whose side is .5 (of a linear unit) has an area of .25 (of a square unit). Show this by diagram.

2. Answer the following and illustrate by diagram if necessary:

$.1^2 = ?$	$.2^2 = ?$	$.3^2 = ?$	$.4^2 = ?$
$.5^2 = ?$	$.6^2 = ?$	$.7^2 = ?$	$.8^2 = ?$
$1.2^2 = ?$	$1.5^2 = ?$	$1.6^2 = ?$	$1.8^2 = ?$

(a) Find the sum of the twelve results.

3. A square of sheet brass whose edge is .9 of a foot is what part of a square foot?

301. The Square Root of Decimals.

1. The square root of .25 is — —.

Note 1.—A square whose area is .25 (of a square unit) is .5 (of a linear unit) in length. Show this by diagram.

Note 2.—Only those decimals are perfect squares which, when in their lowest decimal terms, have numerators that are perfect squares and denominators that are perfect squares The decimal denominators that are perfect squares are 100, 10000, 1000000, etc.

2. What is the square root of $\frac{49}{100}$? Of .36? Of .64?

$\sqrt{\frac{144}{100}} = ?$ $\sqrt{1.44} = ?$ $\sqrt{2.25} = ?$ $\sqrt{6.25} = ?$

(b) Find the sum of the seven results.

3. How long is the edge of a square of zinc whose area is 4.84 square feet? *

4. What is the perimeter of a piece of land whose area is .49 of a square mile?

*4.84 feet is $\frac{121}{25}$ feet.

Powers and Roots.

302. A product obtained by using a number three times as a factor is called **the third power,** or **the cube** of the number; thus, 125, $(5 \times 5 \times 5)$, is the third power, or the cube of 5.

NOTE.—One hundred twenty-five is called the *third* power of 5, because it may be obtained by using 5 *three* times as a factor. It is called the *cube* of 5, because it is the number of *cubic units* in a cube whose edge is 5 linear units.

$1^3 = 1$ $2^3 = 8$ $3^3 = 27$ $4^3 = 64$ $5^3 = 125$

$6^3 = 216$ $7^3 = 343$ $8^3 = 512$ $9^3 = 729$ $10^3 = 1000$

1. Find the cube of 12; of 13; of 14; of 15.

$16^3 = ?$ $17^3 = ?$ $18^3 = ?$ $19^3 = ?$ $20^3 = ?$

(a) Find the sum of the nine results.

303. The **cube root** of a number is one of its three equal factors.

The radical sign with a figure 3 over it, indicates that the cube root of the number following it, is to be taken; thus, $\sqrt[3]{512}$, means, the cube root of 512.

RULE.—*To find the cube root of an integral number that is a perfect cube, resolve the number into its prime factors and take one third of them as factors of the root; that is, one third as many 2's, 3's, or 5's, etc., as there are 2's, 3's, or 5's in the factors of the number.*

1. Find the cube root of 216.

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$. $\sqrt[3]{216} = 2 \times 3$

2. Find the cube root of 1728; of 3375; of 2744; of 10648; of 5832.

(b) Find the sum of the five results.

Powers and Roots.

304. Miscellaneous Problems.

1. Square 42. Then resolve the square of 42 into its prime factors and compare them with the prime factors of 42.

2. Cube 42. Then resolve the cube of 42 into its prime factors and compare them with the prime factors of 42.

3. Square 45. Then resolve the square of 45 into its prime factors and compare them with the prime factors of 45.

4. Cube 45. Then resolve the cube of 45 into its prime factors and compare them with the prime factors of 45.

5. Square 27. Then resolve the square of 27 into its prime factors and compare them with the prime factors of 27.

6. Cube 27. Then resolve the cube of 27 into its prime factors and compare them with the prime factors of 27.

7. Divide the cube of 15 by the square of 15.

8. Divide the cube of $\frac{3}{4}$ by the square of $\frac{3}{4}$.

9. Divide the cube of .7 by the square of .7.

10. Divide the cube of 2.5 by the square of 2.5.

11. Find the square root of each of the following perfect squares:

(1) 3025	(2) 4225	(3) 5625	(4) 7225
(5) $\frac{144}{625}$	(6) $\frac{256}{676}$	(7) $\frac{49}{324}$	(8) $\frac{169}{529}$
(9) .64	(10) .0064	(11) .0625	(12) 2.56 *

12. Find the cube root of each of the following perfect cubes:

(1) 1728	(2) 15625	(3) 3375	(4) 9261
(5) $\frac{8}{125}$	(6) $\frac{27}{512}$	(7) $\frac{64}{343}$	(8) $\frac{1}{729}$

* Think of this number as $\frac{256}{100}$.

Algebra.

305. To Find the Square Root of Numbers Represented by Letters and Figures.

Explanation.

Since the square root of a number is one of its two equal factors the square root of a^4, $(a \times a \times a \times a)$, is a^2, $(a \times a)$. The square root of a^2 is a. The square root of a^6 is a^3. Let $a = 3$, and verify each of the foregoing statements.

1. $\sqrt{b^4} = ?$ $\sqrt{b^6} = ?$ $\sqrt{b^8} = ?$ Verify.

2. $\sqrt{a^2b^2} = ?$ $\sqrt{a^2b^4} = ?$ $\sqrt{a^4b^4} = ?$ Verify.

3. $\sqrt{4\,a^2b^2} = ?$* $\sqrt{9\,a^4b^2} = ?$ $\sqrt{16\,a^2b^4} = ?$

4. $\sqrt{25\,x^2y^2} = ?$ $\sqrt{36\,x^2y^4} = ?$ $\sqrt{49\,x^4y^4} = ?$

5. Let $a = 2$, $b = 3$, and $c = 5$, and find the numerical value of each of the following:

(1) $\sqrt{a^2b^4}$ (2) $\sqrt{a^4c^2}$ (3) $\sqrt{b^2c^4}$

(4) $\sqrt{a^4b^4}$ (5) $\sqrt{a^2b^2c^2}$ (6) $\sqrt{a^2b^4c^2}$

(a) Find the sum of the six results.

6. Let $a = 2$, $b = 3$, $x = 5$, and $y = 7$, and find the numerical value of each of the following:

(1) $a\sqrt{x^2y^2}$ (2) $b\sqrt{x^4y^2}$ (3) $a\sqrt{x^4y^4}$

(4) $3\,a\sqrt{x^4}$ (5) $4\,b\sqrt{y^4}$ (6) $5\,a\sqrt{x^4y^4}$

(b) Find the sum of the six results.

* The factors of $4\,a^2b^2$ are 2, 2, a, a, b, b.

Algebra.

306. To Find the Cube Root of Numbers Represented by Letters and Figures.

EXPLANATION.

Since the cube root of a number is one of its three equal factors, the cube root of a^6, $(a \times a \times a \times a \times a \times a)$, is a^2, $(a \times a)$. The cube root of a^3 is a. The cube root of a^9 is a^3. Let $a = 2$, and verify each of the foregoing statements.

PROBLEMS.

1. $\sqrt[3]{b^6} = ?$ $\sqrt[3]{b^3} = ?$ $\sqrt[3]{b^9} = ?$ Verify.

2. $\sqrt[3]{a^3b^3} = ?$ $\sqrt[3]{a^6b^3} = ?$ $\sqrt[3]{a^6b^6} = ?$ Verify.

3. $\sqrt[3]{8a^6} = ?$ $\sqrt[3]{27\,b^3} = ?$ $\sqrt[3]{64a^6} = ?$ Verify.

4. Let $a = 2$, $b = 3$, and $c = 5$, and find the numerical values of each of the following:

(1) $\sqrt[3]{a^3b^6}$ (2) $\sqrt[3]{8\,c^3}$ (3) $\sqrt[3]{27\,b^3}$

(4) $2\sqrt[3]{a^6b^3}$ (5) $3\sqrt[3]{b^3c^3}$ (6) $4\sqrt[3]{b^6c^3}$

(a) Find the sum of the six results.

307. Miscellaneous Problems.

Let $a = 2$, $b = 3$, $x = 5$, and $y = 7$, and find the numerical value of each of the following:

(1) $ab + \sqrt{x^2y^4}$ (2) $ab\sqrt{x^2y^4}$ (3) $2a\sqrt{x^4y^4}$

(4) $ab + \sqrt[3]{x^3y^6}$ (5) $ab\sqrt[3]{x^3y^6}$ (6) $2b\sqrt[3]{x^6y^6}$

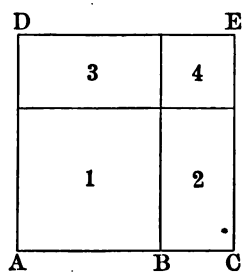
(7) $\frac{1}{4}\sqrt{a^4b^2}$ (8) $\sqrt{\frac{1}{4}a^4b^2}$* (9) $\frac{3}{4}\sqrt{a^4x^2}$

(b) Find the sum of the nine results.

* The factors of this number are $\frac{1}{2}$, $\frac{1}{2}$, a, a, a, a, b, b.

Geometry.

308. The Square of the Sum of Two Lines.



1. Study the diagram and *observe*—

(1) That the line AC is the sum of the lines AB and BC.

(2) That the square, 1, is the square of AB.

(3) That the rectangle, 2, is as long as AB and as wide as BC.

(4) That the rectangle, 3, is as long as AB and as wide as BC.

(5) That the square, 4, is the square of BC.

(6) That the square, ACED, is the square of the sum of AB and BC.

2. Since a similar diagram may be drawn with any two lines as a base, the following general statement may be made:

The square of the sum of two lines is equivalent to the square of the first plus twice the rectangle of the two lines plus the square of the second.

3. If the line AB is 10 inches and the line BC, 5 inches, how many square inches in each part of the diagram and how many in the sum of the parts?

4. Consider the line AB 10 inches and the line BC 3 inches and find the area of each part of the diagram.

5. Suppose the line AB is equal to the line BC; what is the shape of 2 and 3?

6. In the light of the above diagram study the following:

$14^2 = 196$. $(10 + 4)^2 = 10^2 + 2\ (10 \times 4) + 4^2 = 196$.

$16^2 = ?$ $(10 + 6)^2 =$ _____

$25^2 = ?$ $(20 + 5)^2 =$ _____

309. Miscellaneous Review.

1. What is the square root of $a^2 b^2$?
   What is the square root of $3 \times 3 \times 5 \times 5$?

2. What is the cube root of $a^3 b^3$?
   What is the cube root of $2 \times 2 \times 2 \times 7 \times 7 \times 7$?

3. What is the square root of $a^2 b^4$?
   What is the square root of $5^2 \times 3^4$?

4. What is the cube root of $a^6 b^3$?
   What is the cube root of $3^6 \times 5^6$?

5. The area of a certain square floor is 784 square feet. How many feet in the perimeter of the floor?

6. The area of a certain square field is 40 acres. How many rods of fence will be required to enclose it?

7. The solid content of a certain cube is 216 cubic inches. How many square inches in one of its faces?

8. If there are 64 square inches in one face of a cube, how many cubic inches in its solid content?

9. The square of $(30 + 5)$ is how many more than the square of 30 plus the square of 5?

10. The square of $(40 + 3)$ is how many more than the square of 40 plus the square of 3?

11. The square of a is a^2; the square of $2a$ is $4a^2$. The square of two times a number is equal to how many times the square of the number itself?

12. The square of an 8-inch line equals how many times the square of a 4-inch line?

13. The square of a 6-inch line equals how many times the square of a 2-inch line?

SQUARE ROOT.

310. To Find the Approximate Square Root of Numbers that are not Perfect Squares.

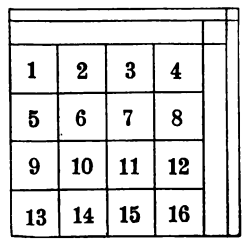
Find the square root of 1795.

Regard the number as representing 1795 1-inch squares. These are to be arranged in the form of a square, and the length of its side noted.

100 1-inch squares = 1 10-inch square.

1700 1-inch squares = 17 10-inch squares.

But 16 of the 17 10-inch squares, can be arranged in a square that is 4 by 4; that is, 40 inches by 40 inches. See diagram.



After making this square (40 inches by 40 inches) there are (1700 — 1600 + 95) 195, 1-inch squares remaining. From these, additions are to be made to two sides of the square already formed. Each side is 40 inches; hence the additions must be made upon a base line of 80 inches. These additions can be as many inches wide as 80 is contained times in 195.* 195 ÷ 80 = 2 +. The additions are 2 inches wide. These will require 2 times 80, + 2 times 2, = 164 square inches.

After making this square (42 in. by 42 in.) there are (195 — 164) 31 square inches remaining. If further additions are to be made to the square, the 31 square inches must be changed to tenth-inch squares. In each 1-inch square there are 100 tenth-inch squares; in 31 square inches there are 3100 tenth-inch squares. From these, additions are to be made upon two sides of the 42-inch square. 42 inches equal 420 tenth-inches. The additions must be made upon a base line (420 × 2) 840 tenth-inches long. These additions can be as many tenth-inches wide as 840 is contained times in 3100. 3100 ÷ 840 = 3+. The additions are 3 tenth-inches wide. These will require 3 times 840, + 3 times 3, = 2529 tenth-inch squares.

After making this square (42.3 by 42.3) there are (3100 — 2529) 571 tenth-inch squares remaining. (If further additions are to be made to the square, the 571 tenth-inch squares must be changed to hundredth-inch squares.) The square root of 1795, true to tenths, is 42.3.

* Allowance must be made for filling the little square shown at the upper right hand corner of the diagram.

Square Root.

NOTE.—Pupils who have mastered the work on the preceding page will have no difficulty in discovering that the same result may be obtained by the following process:

Find the square root of 1795.

Operation.

$$
\begin{array}{rr|l}
 & & \overset{\frown}{17}\overset{\frown}{95}.^{\vee}(42.36 \\
 & & 16 \\
\hline
40 \times 2 = & 80\,* & 195 \\
 & 2 & 164 \\
\hline
420 \times 2 = & 840 & 3100 \\
 & 3 & 2529 \\
\hline
4230 \times 2 = & 8460 & 57100 \\
 & 6 & 50796 \\
\hline
 & & 6304
\end{array}
$$

Rule.

1. *Beginning with the decimal point, group the figures as far as possible into periods of two figures each.*

2. *Find the largest square in the left-hand period* † *and place its root at the right as the first figure of the complete root.*

3. *Subtract the square from the left-hand period and to the difference annex the next period. Regard this as a dividend.*

4. *Take 2 times 10 times the root already found as a trial divisor, and find how many times it is contained in the dividend. Write the quotient as the second figure of the root, and also as a part of the divisor. Multiply the entire divisor by the second figure of the root, subtract the product from the dividend and proceed as before.*

NOTE.—If, in applying the foregoing rule, a dividend is found that will not contain the divisor, annex a zero to the root, a zero to the trial divisor, a new period to the dividend, and proceed as before.

PROBLEMS.

Find the approximate square root (true to tenths):

(1) 875.	(2) 1526.	(3) 2754.	(4) 4150.
(5) 624.	(6) 624.7	(7) 62.47.	(8) 6.24.

(a) Find the sum of the eight results.

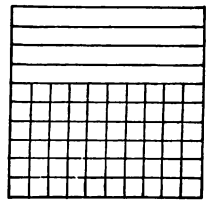
* The entire divisor is 80 and 2; that is, 82.

† The left-hand period may consist of either one or two figures.

Square Root.

311. To Find the Approximate Square Root of Decimals That Are Not Perfect Squares.

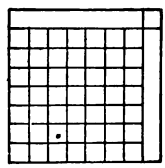
Find the square root of .6



Regard the number as representing .6 of a 1-inch square. .6 of a 1-inch square = 60 tenth-inch squares.

But 49 of the 60 tenth-inch squares can be arranged in a square that is 7 by 7; that is, 7 tenths of an inch by 7 tenths of an inch.

After making this square there are (60 — 49) 11 tenth-inch squares remaining. If additions are to be made to the square, the 11 tenth-inch squares must be changed to hundredth-inch squares. In each tenth-inch square there are 100 hundredth-inch squares; in 11 tenth-inch squares there are 1100 hundredth-inch squares. From these, additions are to be made upon two sides of the .7-inch square. .7 = 70 hundredths. The additions must be made upon a base line (70 × 2) 140 hundredth-inches long. These additions can be as many hundredth-inches wide as 140 is contained times in 1100. 1100 ÷ 140 = 7 +. The additions are 7 hundredth-inches wide. These will require 7 times 140, + 7 times 7, = 1029 hundredth-inch squares.



After making this square (.77 by .77) there are (1100 — 1029) 71 hundredth-inch squares remaining. (If further additions are to be made to the square the 77 hundredth-inch squares must be changed to thousandth-inch squares.) The square root of .6, true to hundredths, is .77.

Note.—The work on this page should be first presented orally by the teacher. It must be given very slowly. Great care must be taken that pupils image each magnitude when its word-symbol is spoken by the teacher. Any attempt to move forward more rapidly than this can be done by the slowest pupil, will result in failure so far as that pupil is concerned. The great skill of many pupils in the Illinois School for the Blind in such work as this, is to be attributed mainly to their practice in creating imaginary magnitudes. When the teacher says *tenth-inch square*, they "think a tenth-inch square." Its image comes immediately into consciousness. Teachers of pupils who have sight, may obtain invaluable suggestion from the mathematical ability of these pupils who have opportunity for comparatively little sense-perception of magnitudes, but who, from necessity, are constantly trained to "see with the mind's eye." See preface of this book, page 3.

Square Root.

NOTE.—Pupils who have mastered the work on the preceding page will readily understand the following process. See rule on page 212.

1. Find the square root of .6.

Operation.

$$
\begin{array}{rr|l}
 & & .\overset{\vee}{6}0\overset{\frown}{00}(.774 \\
 & & 49 \\
\hline
70 \times 2 = & 140 & 1100 \\
 & 7 & 1029 \\
\hline
770 \times 2 = & 1540 & 7100 \\
 & 4 & 6176 \\
\hline
 & & 924
\end{array}
$$

Observe—

1. *That in grouping decimals for the purpose of extracting the square root it is necessary to begin at the decimal point.*

2. *That the square root of any number of hundredths is a number of tenths; the square root of any number of ten-thousandths is a number of hundredths, etc.*

2. Find the square root of 54264.25.

$$
\begin{array}{rr|l}
 & & \overset{\frown}{5}\overset{\frown}{4}\overset{\frown}{2}\overset{\frown}{6}4.\overset{\vee}{2}\overset{\frown}{5}(232.946 \\
 & & 4 \\
\hline
20 \times 2 = & 40 & 142 \\
 & 3 & 129 \\
\hline
230 \times 2 = & 460 & 1364 \\
 & 2 & 924 \\
\hline
2320 \times 2 = & 4640 & 440.25 \\
 & 9 & 418.41 \\
\hline
23290 \times 2 = & 46580 & 21.8400 \\
 & 4 & 18.6336 \\
\hline
232940 \times 2 = & 465880 & 3.206400 \\
 & 6 & 2.795316 \\
\hline
 & & .411084
\end{array}
$$

Observe that the trial divisor is always 2 times 10 times the part of the root already found.

Square Root.

312. The following numbers are perfect squares. Find their square roots by both the factor method and the method given on the four preceding pages.

(1) 6889 (2) 841 (3) 71824

(4) 1849 (5) 729 (6) 60516

(a) Find the sum of the six results.

(7) $\frac{841}{625}$ (8) $\frac{289}{225}$ (9) $\frac{576}{625}$

(10) $\frac{4489}{625}$ (11) $\frac{2916}{625}$ (12) $\frac{900}{625}$

(b) Find the sum of the six results.

(13) .81 (14) .0625 (15) .04

(16) 1.21 (17) .7921 (18) .0004

(c) Find the sum of the six results.

313. Miscellaneous.

1. The square of a number represented by one digit gives a number represented by —— or —— digits.

2. The square of a number represented by two digits gives a number represented by —— or —— digits.

3. The square of a number represented by three digits gives a number represented by —— or —— digits.

4. The square root of a perfect square represented by one or two digits is a number represented by ——— digit.

5. The square root of a perfect square represented by three or four digits is a number represented by —— —— digits.

6. The square root of a perfect square represented by five or six digits is a number represented by ——— digits.

Square Root.

314. Miscellaneous Problems.

1. What is one of the two equal factors of 9216?

2. What is one of the four equal factors of 20736?*

3. If a body of 7921 soldiers were arranged in a solid square, how many soldiers would there be on each side?

4. How many rods of fence will enclose a square field whose area is 40 acres?

5. How many rods long is one side of a square piece of land containing exactly one acre?†

6. If the surface of a cubical block is 150 square inches, what is the length of one edge of the cube?

7. How many rods of fence will enclose a square piece of land containing 4 acres 144 square rods?

8. Find the side of a square equal in area to a rectangle that is 15 ft. by 60 ft.

9. Compare the amount of fence required to enclose two fields each containing 10 acres: one field is square, and the other is 50 rods long and ——— rods wide.

10. Find the area of the largest possible rectangle having a perimeter of 40 feet.

11. If a square piece of land is $\frac{1}{4}$ of a square mile, how much fence will be required to enclose it?

12. Find the squares of numbers from 10 to 20, inclusive, and memorize them.

*To find one of the four equal factors of a number (the 4th root) extract the square root of the square root. Why? What is the fourth root of 81?

†Find the answer to problem 5, true to hundredths of a rod.

Algebra.

315. Square Root and Area.

1. If a piece of land containing 768 square rods is three times as long as it is wide, how wide is it? *

Let x = the width,
then $3x$ = the length,
and $x(3x)$ or $3x^2$ = the area.

$$3x^2 = 768$$
$$x^2 = 256$$
$$x = 16$$

2. If a certain room is twice as long as it is wide, and the area of the floor 968 square feet, what is the length and the breadth of the room?

3. One half of the length of Mr. Smith's farm is equal to its breadth. The farm contains 80 acres. How many rods of fence will be required to enclose it?

4. Each of four of the faces of a square prism is an oblong whose length is twice its breadth. The area of one of these oblongs is 72 square inches. What is the solid content of the prism.†

5. The width of a certain field is to its length as 2 to 3. Its area is 600 square rods. The perimeter of the field is how many rods?

6. If $\frac{2}{3}$ of the length of an oblong equals the width and its area is 768 square inches, what is the length of the oblong?

7. If to $2\frac{1}{2}$ times the square of a number you add 15 the sum is 375. What is the number?

*To solve this problem arithmetically, one must discern that this piece of land can be divided into three equal squares, the side of each square being equal to the width of the piece.

†Let no pupil attempt to solve this problem without first bringing into consciousness an image of the prism.

Algebra.

316. Square Root and Proportion.

When the same number forms the second and the third term of a proportion it is called a **mean proportional,** of the first and the fourth term; thus, in the proportion 3 : 6 : : 6 : 12, 6 is a mean proportional of 3 and 12.

Example.

In the proportion $12 : x : : x : 75$, find the value of x.

Since the product of the means equals the product of the extremes, x times x equals 12 times 75, or,

$$x^2 = 900.$$
$$x = 30.$$

Find the value of x in each of the following proportions:

1. $9 : x : : x : 16$.
2. $16 : x : : x : 25$.
3. $8 : x : : x : 32$.
4. $12 : x : : x : 48$.
5. $5 : x : : x : 125$.
6. $36 : x : : x : 49$.

(a) Find the sum of the six mean proportionals.

7. An estate was to be divided so that the ratio of A's part to B's would equal the ratio of B's part to C's. If A received $8000 and C received $18000, how much should B receive?

8. Find the mean proportional of $\frac{2}{5}$ and $1\frac{3}{5}$.

9. The ratio of the areas of two squares is as 4 to 9. What is the ratio of their lengths?

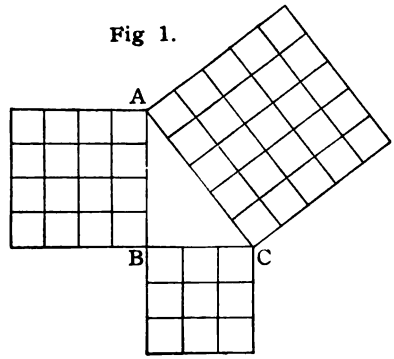
10. The ratio of the lengths of two squares is as 9 to 16. What is the ratio of their areas?

11. The area of the face of one cube is to the area of the face of another cube as 16 to 25. What is the ratio of the solid contents of the cubes?

317. RIGHT TRIANGLES.

1. The longest side of a right-triangle is the **hypothenuse**. Either of the other sides may be regarded as the **base** and the remaining side as the **perpendicular**.

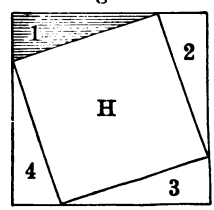
Fig 1.



In Fig. 1, AC is the ——— ; BC, the ———, and AB the ———.

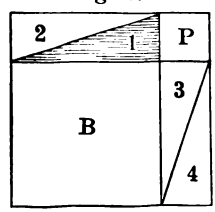
2. Convince yourself by examination of the figures here given and by careful measurements and paper cutting, that the square of the hypothenuse of a right-triangle is equivalent to the sum of the squares of the other two sides.

Fig. 2.



Figures 2 and 3 are equal squares. If from figure 2, the four right-triangles, 1, 2, 3, 4, be taken, H, the square of the hypothenuse, remains. If from figure 3, the four right-triangles (equal to the four right-triangles in figure 2) be taken, B, the square of the base, and P, the square of the perpendicular, remain. When equals are taken from equals the remainders are equal, therefore the square, H, equals the sum of the squares B and P.

Fig 3.



3. To find the hypothenuse of a right triangle when the base and perpendicular are given: *Square the base; square the perpendicular; extract the square root of the sum of these squares.*

4. Give a rule for finding the perpendicular when the base and hypothenuse are given.

318. Miscellaneous Review.

1. Find approximately the diagonal of a square whose side is 20 feet.*

2. Find approximately the distance diagonally across a rectangular floor, the length of the floor being 30 feet and its breadth 20 feet.

3. How long a ladder is required to reach to a window 25 feet high if the foot of the ladder is 6 feet from the building and the ground about the building level?

4. If the length of a rectangle is a, and its breadth, b, what is the diagonal?

5. The base of a right triangle is 40 rods and its perpendicular, 60 rods. (a) What is its hypothenuse? (b) What is its area? (c) What is its perimeter?

6. The area of a certain square piece of land is $2\frac{1}{2}$ acres. (a) Find (in rods) its side. (b) Find its perimeter. (c) Find its diagonal, true to tenths of a rod.

7. The length of a rectangular piece of land is to its breadth as 4 to 3. Its area is 30 acres. (a) Find its breadth. (b) Find its perimeter. (c) Find the distance diagonally across it.

8. A certain piece of land is in the shape of a right-triangle. Its base is to its altitude as 3 to 4. Its area is 96 square rods. (a) Find the base. (b) Find the altitude. (c) Find the perimeter.

9. Find one of the two equal factors of 93025.

10. Find one of the three equal factors of 74088.

* From the study of right-triangles on page 219 it may be learned that the diagonal of a square is equal to the square root of twice the square of its side.

METRIC SYSTEM.

NOTE.—Work equivalent to that found on pp. 154, 164, 174, etc., of the Werner Arithmetic, Book II, should be done by the pupils before this chapter is attempted. If such work has not been done the teacher should present the subject orally in the order given on the pages named. Pupils must learn to think of quantity in metric units.

319. All units in the metric system of measures and weights are derived from the primary unit known as the **meter.**

When the length of the primary unit of this system was determined it was supposed to be one ten-millionth of the distance from the equator to the pole. A pendulum that vibrates seconds is nearly one meter long.*

In the names of the derived units of this system the prefix *deka* means 10; *hekto* means 100; *kilo* means 1000; *myria* means 10000; *deci* means tenth; *centi* means hundredth; *milli* means thousandth.

320. LINEAR MEASURE.

10 millimeters (mm.)	= 1 centimeter (cm.).†
10 centimeters	= 1 decimeter (dm.).
10 decimeters	= 1 meter (m).
10 meters	= 1 dekameter (Dm).
10 dekameters	= 1 hektometer (Hm.).
10 hektometers	= 1 kilometer (Km.).
10 kilometers	= 1 myriameter (Mm.).

* The teacher must see that a meter stick is provided and that the pupils use it in measurement until they can think its multiples and its divisors without reference to other units of measurement.

† In the common pronunciation of these words the primary accent is on the first syllable and a secondary accent on the penultimate syllable; thus, cen'timé-ter. In the better pronunciation the accent is on the vowel preceding the letter m, that is, on the antepenultimate syllable: thus, centim'eter, dekam'eter, etc.

Metric System.

321. The names of the units of **surface measurement** are the same as those used for linear measurement, combined with the word *square;* thus, a surface equivalent to a square whose side is a meter, is 1 **square meter.**

The pupil, if properly taught to this point, will be able, without difficulty, to fill the blanks in the table of—

SQUARE MEASURE.

100 square millimeters (sq. mm.)	= 1 square centimeter (sq. cm.).
——— square centimeters	= 1 square decimeter (sq. dm.).
——— square decimeters	= 1 square meter (sq. m.).
——— square meters	= 1 square dekameter (sq. Dm.).
——— square dekameters	= 1 square hektometer (sq. Hm.).
——— square hektometers	= 1 square kilometer (sq. Km.).
——— square kilometers	= 1 square myriameter (sq. Mm.).

NOTE.—The special unit of surface measure for measuring land is equivalent to a square whose side is ten meters. This unit is called an **ar.**

100 centars (ca.) = 1 ar (a.).
100 ars = 1 hektar (Ha.).

EXERCISE.

1. In a square decimeter there are ——— sq. cm.
2. In 2 square decimeters there are ——— sq. cm.
3. In a 2-decimeter square there are ——— sq. cm.
4. In a square meter there are ——— sq. dm.
5. In 2 square meters there are ——— sq. dm.
6. In a 2-meter square there are ——— sq. dm.
7. In a square meter there are ——— sq. cm.
8. In 2 square meters there are ——— sq. cm.
9. In a 2-meter square there are ——— sq. cm.

Metric System.

322. The names of the units of **volume measurement** are the same as those used for linear measurement, combined with the word *cubic;* thus, a volume equivalent to a cube whose edge is a meter, is 1 **cubic meter.**

The pupil should be able easily to fill the blanks in the table of—

VOLUME MEASURE.

1000 cubic millimeters (cu. mm.)	= 1 cubic centimeter (cu. cm.).*
—— cubic centimeters	= 1 cubic decimeter (cu. dm.)
—— cubic decimeters	= 1 cubic meter (cu. m.).
—— cubic meters	= 1 cubic dekameter (cu. Dm.).
—— cubic dekameters	= 1 cubic hektometer (cu. Hm.)

NOTE 1.—The special unit of capacity, for measuring liquids, grain, small fruits, etc., is the **liter.** It is equal to 1 cubic decimeter. 10 liters (l.) = 1 dekaliter (Dl.), and 1 tenth of a liter = 1 deciliter (dl.), etc.

NOTE 2.—The special unit for measuring wood is the **ster.** It is equal to 1 cubic meter.

EXERCISE.

1. A cubic meter equals —— liters.
2. A cubic meter equals —— cubic decimeters.
3. A cubic meter equals —— cubic centimeters.
4. A cubic decimeter equals —— cubic centimeters.
5. Two cubic decimeters equal —— cubic centimeters.
6. A 2-decimeter cube equals —— cubic centimeters.
7. A 5-centimeter square equals —— square centimeters.
8. A 5-centimeter cube equals —— cubic centimeters.
9. One tenth of a liter equals —— cubic centimeters.
10. One deciliter equals —— cubic centimeters.
11. One dekaliter equals —— cubic decimeters.

* The abbreviation cc. is often used for cubic centimeter.

Metric System.

323. The primary unit of weight is the **gram.** This equals the weight of one cubic centimeter of pure water.

WEIGHT.

10 milligrams (mg.)	= 1 centigram (cg.).
10 centigrams	= 1 decigram (dg.).
10 decigrams	= 1 gram (g.).
10 grams	= 1 dekagram (Dg.).
10 dekagrams	= 1 hektogram (Hg.).
10 hektograms	= 1 kilogram (Kg.).

NOTE.—The special unit for the weight of very heavy articles is the **tonneau.** It equals the weight of a cubic meter of pure water, or 1000 kilograms.

EXERCISE.

1. The weight of 1 liter of water is —— grams.
2. The weight of 6 cubic centimeters of water is ——.
3. The weight of a cubic decimeter of water is ——.

4. One kilogram of water equals —— cubic centimeters.
5. One hektogram of water equals —— cc.
6. One dekagram of water equals —— cubic centimeters.

7. If the specific gravity of iron is 7.5, what is the weight of a cubic centimeter of iron?
8. If the specific gravity of cork is $\frac{1}{4}$, what is the weight of a cubic decimeter of cork?
9. If the specific gravity of oil is .9, what is the weight of a liter of oil?

10. What is the weight of a cubic meter of stone whose specific gravity is 2.5?
11. What is the weight of a cubic decimeter of wood whose specific gravity is .8?

Metric System.

324. Miscellaneous Problems.*

1. Find the area of a rectangular surface that is 1 meter long and 6 decimeters wide. Make a diagram of this surface upon the blackboard.

2. Find the area of a rectangular surface that is 2 decimeters long and 5 centimeters wide. Make a diagram of this surface on your slate or paper.

3. Find the solid content of a 5-centimeter cube. A 5-centimeter cube is what part of a cubic decimeter?

4. Find the solid content of a 4-decimeter cube. A 4-decimeter cube is what part of a cubic meter?

5. Find the entire surface of a 4-centimeter cube. The surface of a 4-centimeter cube is what part of a square decimeter?

6. Find the area of a rectangular surface that is 2.4 yards by 5 yards; of a rectangular surface that is 2.4 meters by 5 meters.

7. Which is the larger of the two surfaces described in problem 6?

8. Find the area of a rectangular surface that is 3.5 yards by 2.5 yards; of a rectangular surface that is 3.5 meters by 2.5 meters.

9. Find the volume of a rectangular solid that is 3.4 feet by 3 feet by 2 feet; of a rectangular solid that is 3.4 meters by 3 meters by 2 meters.

10. Find the volume of a rectangular solid that is 3.5 meters by 2.3 meters by 4.6 meters.

11. What is the weight of a cubic decimeter of wood whose specific gravity is .5?

* Pupils should draw or image every magnitude whose dimensions are given on this page.

Metric System.

325. Miscellaneous Problems.

1. Estimate in meters the width of the lot upon which the school building stands. Measure it.

2. Estimate in centimeters the width of your desk. Measure it.

3. Estimate in square centimeters the area of a sheet of paper. Measure and compute.

4. Estimate in square meters the area of the blackboard. Measure and compute.

5. Estimate the number of cubic meters of air in the school room. Measure and compute.

6. Estimate in grams the weight of a teaspoonful of water. Weigh it.*

7. Estimate in kilograms your own weight.

8. Estimate in liters the capacity of a water pail.

9. Estimate in kilograms the weight of a gallon of water.

326. Table of Equivalents.

Meter	a little more than 1 yard. . . .	39.37 inches.
Kilometer . . .	nearly $\frac{5}{8}$ of a mile	3280.8 + feet.
Decimeter . . .	nearly 4 inches	3.937 inches.
Ar	nearly $\frac{1}{40}$ of an acre	3.954 sq. rd.
Ster	a little more than $\frac{1}{4}$ cord . . .	35.3 + cu. ft.
Liter	a little more than 1 liquid quart	1.056 + qt.
Gram	nearly $15\frac{1}{2}$ grains	15.4 + grains.
Kilogram . . .	nearly $2\frac{1}{4}$ pounds	2 204 + lb.

Note 1.—A "nickel" is 2 centimeters in diameter, 2 millimeters in thickness, and weighs 5 grams.

Note 2.—A silver dollar weighs about 25 grams.

* Every school should be provided with scales, weights, and measures.

Algebra.

327. Metric Units in Algebraic Problems.

1. I am thinking of a rectangular surface. Its length is 5 times its breadth. Its area is 45 square decimeters. How long and how wide is the surface?

Note.—Let x = the number of decimeters in the breadth of the surface.

2. I am thinking of a triangular surface. Its base is three times its altitude. Its area is 8.64 square meters. What is the length of its base?

3. I am thinking of a cube whose entire surface is 150 square centimeters. What is the length of one of its edges?

4. The perimeter of a certain rectangle is 20.4 meters. Its length is twice its breadth. (a) Find its length and breadth. (b) Find its area.

5. The difference in the weight of two lead balls is 24 grams. The united weight of the two balls is 1 kilogram. (a) Find the weight of each ball. (b) Does the heavier ball weigh more or less than 1 pound?

6. A merchant had three pieces of lace. In the second piece there were twice as many meters as in the first. In the third piece there were 6 meters more than in the second. In the three pieces there were 106 meters. (a) How many meters in each piece? (b) Were there more or less than 43 yards in the second piece?

7. John weighs 3.6 kilograms more than Henry. Together they weigh 83.6 kilograms. (a) Find the weight of each boy. (b) Does John weigh more or less than 90 pounds?

Algebra.

328. Metric Units in Algebraic Problems.

1. A ball rolling down a perfectly smooth and uniformly inclined plane rolls 3 times as far the 2nd second as the 1st; 5 times as far the 3rd second as the 1st; 7 times as far the 4th second as the first. If in 4 seconds it rolls 192 decimeters (a) how far did it roll in the 1st second? (b) in the 4th second? (c) Did it roll more or less than 48 inches in the first second?

2. I am thinking of a right-triangle. Its altitude is to its base as 3 to 4. The sum of its altitude and base is 14 centimeters. (a) Find the altitude. (b) Find the base. (c) Find the area. (d) Find the hypothenuse. (e) Is the hypothenuse more or less than 4 inches?

3. A freely falling body falls three times as far the 2nd second of its fall as it does the 1st second. In two seconds it falls 19.6 meters.* (a) How far does it fall in the 1st second? (b) in the 2nd second?

4. A freely falling body falls 3 times as far the 2nd minute of its fall as it does the 1st minute. In 2 minutes it falls 70560 meters.* (a) How far does it fall in the 1st minute? (b) In the 2nd minute? (c) 70560 meters equals how many kilometers? (d) 70560 meters equals (approximately) how many miles?

5. A freely falling body falls 3 times as far the 2nd half-second as it does the 1st half-second. In one second it falls 4.9 meters. (a) How far does it fall in the 1st half-second? (b) in the 2nd half-second?

*In such problems the resistance of the air is not considered.

Geometry.

329. The Circumference of a Circle.

1. Cut a 3-inch circle from cardboard. By rolling it upon a foot rule, measure its circumference.

2. Measure the diameter of a bicycle wheel; then by rolling it upon the ground or upon the school-room floor, measure its circumference.

3. In a similar manner measure the diameters and the circumferences of other wheels until you are convinced that the circumference of a circle is a little more than —— times its diameter.

4. The circumference of a circle is nearly $3\frac{1}{7}$ times the diameter; more accurately, it is 3.141592+ times the diameter.

Note.—It is a curious fact that the diameter of a circle being given in numbers it is impossible to express in numbers its exact circumference. The circumference being given in numbers it is impossible to express in numbers its exact diameter. In other words, the exact ratio of the circumference to the diameter is not expressible.

5. Find the approximate circumference of a 5-inch circle; of a 7-inch circle; of a 10-inch circle. *

6. Find the approximate diameter of a circle that is 6 ft. in circumference. *

7. The circumference of a 6-inch circle is how many times the circumference of a 3-inch circle?

8. The diameter of a circle whose circumference is 12 inches is what part of the diameter of a circle whose circumference is 24 inches?

* In the solution of such problems as these the pupil may use, as the approximate ratio of the circumference to the diameter, 3.14.

330. Miscellaneous Review.

1. Find the approximate circumference of a circle whose diameter is 3.4 meters.

2. Find the approximate diameter of a circle whose circumference is 3.4 decimeters.

3. Find the approximate diameter in yards of a circular 1-mile race track; of a half-mile track.

4. Find the approximate diameter in meters of a circular 1-kilometer race track; of a half-kilometer track.

5. A 28-inch bicycle wheel will make how many revolutions in running one mile?

6. A 70-centimeter bicycle wheel will make how many revolutions in running 1 kilometer? *

See table on page 226 and give approximate answers to the following:

7. Forty meters are how many yards?
8. Forty yards are how many meters?

9. Forty kilometers are how many miles?
10. Forty miles are how many kilometers?

11. Forty ars are how many acres?
12. Forty acres are how many ars?

13. Forty sters are how many cords?
14. Forty cords are how many sters?

15. Forty liters are how many quarts?
16. Forty quarts are how many liters?

17. Forty kilograms are how many pounds?
18. Forty pounds are how many kilograms?

* The exact answer to such questions as this cannot be found: but the approximation is practically correct.

DENOMINATE NUMBERS.

Linear Measure.

NOTE.—In part to provide for ready reference, and in part to give further application of the principles presented on the preceding pages, the remaining pages of this book are devoted chiefly to denominate numbers.

331. The English and United States standard unit of length is the **Imperial yard** arbitrarily fixed by Act of Parliament and afterward adopted in the United States. It is about $\frac{3}{3}\frac{6}{6}\frac{0}{0}\frac{0}{0}\frac{0}{0}\frac{0}{0}$ of the length of a pendulum that vibrates once a second at the level of the sea in the latitude of London. It is $\frac{9}{1}\frac{1}{0}\frac{4}{0}\frac{4}{0}\frac{0}{7}$ of a meter.

TABLE.

12 inches (in.)	= 1 foot (ft)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
$16\frac{1}{2}$ feet	= 1 rod.
320 rods	= 1 mile (mi.)
1760 yards	= 1 mile.
5280 feet	= 1 mile.

1 fathom (used in measuring the depth of the sea)	= 6 feet.
1 knot (used in navigation)	= 1.15 + miles.
1 league (used in navigation)	= 3 knots.
1 hand (used in measuring the heights of horses)	= 4 inches.
1 chain (used by civil engineers)	= 100 feet.
1 chain (used by land surveyors)	= 66 feet.
1 pace (used in measuring approximately)	= $\frac{1}{2}$ of a rod.
1 barleycorn (used in grading length of shoes)	= $\frac{1}{3}$ of an inch.
1 furlong (a term nearly obsolete)	= $\frac{1}{8}$ of a mile.

Denominate Numbers—Linear Measure.

Exercise.

1. Mont Blanc is 15810 feet, or about —— miles high.
2. Mt. Everest is 29000 feet, or about —— miles high.
3. Commodore Dewey opened fire on the enemy at a distance of 5000 yards, or about —— miles.
4. My horse, measured over the front feet, is 16¼ hands, or —— feet —— inches high.
5. The vessel seemed to be about three leagues, or —— miles distant.
6. On sounding, they found the depth of the water to be 15 fathoms, or —— feet.
7. The cruiser made 20 knots or about —— miles, an hour.
8. The length of the lot was 36 paces, or about —— rods.
9. 10000 feet is nearly —— miles.
10. 15000 feet is nearly —— miles.
11. 1000 yards is about — — of a mile.
12. 100 feet is —— rods —— foot.
13. 200 feet is —— rods —— feet.
14. 300 feet is —— rods —— feet.
15. A kilometer is about —— rods.
16. A Civil Engineer's chain is —— rods —— foot.

Problems.

1. A seven-foot drive wheel of a locomotive makes how many revolutions to the mile?
2. Which is the longest distance, 5 miles 319 rods 16 feet 6 inches, 5 miles 319 rods 5 yards 1 foot 6 inches, or 6 miles?
3. Reduce 40 rd. 4 ft. 5 in. to inches.
4. Reduce 1100 inches to yards, feet, and inches.

Denominate Numbers—Surface Measure.

332. The standard unit of surface measure is a **square yard** which is the equivalent of a 1-yard square.* This unit, like the square foot, square inch, square rod, and square mile, is derived from the corresponding unit of linear measure.

TABLE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.),
9 square feet	= 1 square yard (sq. yd.).
30¼ square yards	= 1 square rod (sq. rd.).
272¼ square feet	= 1 square rod.
160 square rods	= 1 acre (A.).
4840 square yards	= 1 acre.
43560 square feet	= 1 acre.
640 acres	= 1 square mile (sq. mi.).

EXERCISE.

1. Show by a drawing that there are 144 square inches in a 1-foot square.

2. Show by a drawing that there are 9 square feet in a 1-yard square.

3. Show by a drawing that there are 30¼ square yards in a 1-rod square.

4. Estimate the number of square yards of blackboard in the room ; the number of square feet of blackboard.

5. Estimate the number of square feet in the floor of the schoolroom ; the number of square yards.

6. Estimate the square yards of plastering on the walls of the schoolroom.

7. Estimate the number of square rods in the schoolhouse lot. Is the lot more or less than ½ of an acre?

* A surface that is 9 ft. long and 1 ft. wide is a square yard though it is not itself a square. It is the equivalent of a 1-yard square.

Denominate Numbers—Surface Measure.

333. In the measurement of land it is more convenient to use a decimal scale; hence the invention of the **Gunter Chain.** This chain is 4 rods long and is divided into 100 links.

Observe that links are hundredths of chains.
Observe that square chains are tenths of acres.

EXERCISE.

1. Land, 3 chains by 4 chains contains ——— acres.
2. Land, 5 chains by 4 chains contains ——— acres.
3. Land, 3 chains by 8 chains contains ——— acres.
4. Land, 5 chains by 7 chains contains ——— acres.
5. Land, 8 chains by 6 chains contains ——— acres.
6. Two chains 20 links equals ——— chains.
7. Two chains 35 links equals ——— chains.
8. Two chains 75 links equals ——— chains.
9. Two chains 5 links equals ——— chains.
10. Two chains 9 links equals ——— chains.
11. Land, 4 ch. by 4.50 ch. contains ——— acres.
12. Land, 5 ch. by 3.20 ch. contains ——— acres.
13. Make a rule and find the number of acres in each of the following:

(1) Land, 12 chains 35 links by 9 chains 50 links.
(2) Land, 21 chains 8 links by 12 chains 30 links.
(3) Land, 32 chains 25 links by 15 chains 6 links.

(a) Find the sum of the area of the ten pieces of land described on this page.

TO THE TEACHER.—A rod is exactly 25 links. A foot is about $1\frac{1}{4}$ links. Hence rods and feet can be easily changed to chains and links by regarding each 4 rods as 1 chain and each additional rod as 25 links and each additional foot as $1\frac{1}{4}$ links. The error in any one measurement never exceeds 2 inches. 9 rd. 12 ft. = 2 chains 43, (25 + 18), links.

Denominate Numbers—Surface Measure.

334. To determine the amount of **carpet** necessary for a given room several minor problems must be solved which can be best studied by means of an—

EXAMPLE.

1. How many yards of carpet must be purchased for a room 16 ft. by 20 ft. if the carpet is 1 yd. wide?

(1) How many breadths will be necessary if the carpet is put down lengthwise of the room? How much must be cut off or turned under from one breadth in this case?

(2) How many breadths will be necessary if the carpet is put down crosswise of the room? How much must be cut off or turned under from one breadth in this case?

(3) Make two diagrams of the room on a scale of 1 inch to the foot and show the breadths of carpet in each case.

(4) How many yards must be purchased in each case?

(5) If in the first case there is no waste in matching the figure and in the second case there is a waste of 8 inches on each breadth except the first,* which plan of putting down the carpet will require the greater number of yards?

(6) If the carpet costs 90¢ a yard and the conditions are as stated in No. 5, what is the cost of the carpet in each case?

2. How many yards of carpet must be purchased for a room 16 ft. by 20 ft. if the carpet is $\frac{3}{4}$ of a yard wide and there is no waste in matching the figure?

3. How many yards of carpet must be purchased for a room that is 15 ft. 6 in. by 16 ft. 4 in. if the carpet is $\frac{3}{4}$ of a yard wide, is put down lengthwise of the room, and there is no waste in matching the figure?

* Why except the first breadth?

Denominate Numbers.

335. PLASTERING AND PAPERING.

1. How many square yards of plastering in a room (walls and ceiling) that is 15 ft. by 18 ft. and 12 ft. high, an allowance of 12 square yards being made for openings?

NOTE.—In estimating the cost of plastering, allowance is made for "openings" (windows and doors) only when they are very large in proportion to the wall to be covered. Why are plasterers unwilling to deduct the entire area of all the openings?

2. At 24¢ a square yard how much will it cost to plaster a room that is 17 ft. by 20 ft. and 10 feet from the floor to the ceiling, deducting 16 square yards for openings?

3. How many "double rolls" of paper will be required for the walls of a room that is 14 ft. by 16 ft. and 11 ft. high above the baseboards, if an allowance of 1 full "double roll" is made for openings?

NOTE.—Wall paper is usually 18 inches wide. A "single roll" is 24 ft. long. A "double roll" is 48 ft. long. In papering a room 11 ft. high it would be safe to count on 4 full strips from each "double roll." The remnant would be valueless unless it could be used over windows or doors. Since each strip is 18 inches wide, a "double roll" will cover 72 inches (6 ft.) of wall measured horizontally.

4. At 12¢ a "single roll," how much will the paper cost for the walls of a room that is 12 ft. by 14 ft. and 7 ft. above the baseboards, if the area of the openings is equivalent to the surface of 2 "single rolls" of paper?

5. Find the cost, at 25¢ a square yard, of plastering the walls of a room that is 48 ft. by 60 ft. and 18 feet high, deducting 30 square yards for openings.

Denominate Numbers.

336. Farm Problems.

Find how many acres in—

1. A piece of land 1 rod by 160 rods.
2. A piece of land 7 rods by 160 rods.
3. A piece of land 13 rods by 160 rods.
4. A piece of land 22 feet * by 160 rods.
5. A piece of land $8\frac{1}{4}$ yards * by 160 rods.

(a) Find the sum of the five results.

6. A piece of land 8 rods by 80 rods.
7. A piece of land 17 rods by 80 rods.
8. A piece of land $37\frac{1}{2}$ rods by 80 rods.
9. A piece of land $618\frac{3}{4}$ feet * by 80 rods.
10. A piece of land 550 yards * by 80 rods.

(b) Find the sum of the five results.

11. A piece of land 12 rods by 40 rods.
12. A piece of land 27 rods by 40 rods.
13. A piece of land 46 rods by 20 rods.
14. A piece of land 36 rods by 20 rods.
15. A piece of land 264 feet * by 20 rods.

(c) Find the sum of the five results.

16. A piece of land 1 rod by 1 mile.
17. A piece of land 11 rods by 1 mile.
18. A piece of land 66 feet * by 1 mile.
19. A piece of land 99 yards * by 1 mile.
20. A piece of land 198 feet * by $\frac{1}{2}$ of a mile.

(d) Find the sum of the five results.

21. A piece of land $\frac{1}{2}$ of a mile long and as wide as the schoolroom.

* Change to rods.

Denominate Numbers.

337. Farm Problems.

1. A piece of land 1 foot wide and 43560 feet long is how many acres?

2. Change 43560 feet to miles.

3. A piece of land 1 foot wide must be how many miles in length to contain 1 acre?

4. Some country roads are 66 feet wide. How many acres in $8\frac{1}{4}$ miles of such road?

5. How many acres in 1 mile of road that is 4 rods wide?

6. A farmer walking behind a plow that makes a furrow 1 foot wide will travel how far in plowing 1 acre?

7. A farmer walking behind a plow that makes a furrow 16 inches wide will travel how far in plowing 1 acre?

8. If a mowing machine cuts a swath that averages 4 feet in width how far does it move in cutting 1 acre?

9. If potatoes are planted in rows that are 3 feet apart (a) how many miles of row to each acre? (b) How many rods of row to each acre? (c) If 4 rods of row on the average yield 1 bushel, what is the yield per acre?

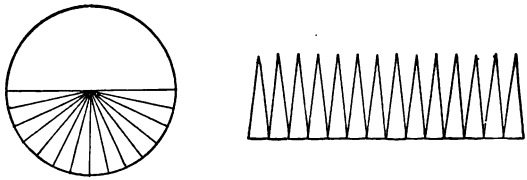
10. Strawberry plants are set in rows that are 2 feet apart. (a) How many miles of row to the acre? (b) How many rods of row to the acre? (c) How many feet of row to the acre?

11. If corn is planted in rows $3\frac{1}{2}$ feet apart and if the "hills" are $3\frac{1}{2}$ feet apart in the row, how many hills to each acre? *

* Think of each "hill" as occupying a piece of land $3\frac{1}{2}$ ft. by $3\frac{1}{2}$ ft.

Geometry.

338. To Find the Area of a Circle.



1. Cut one half of a circular piece of paper as indicated in the diagram.

Observe that if the circle is cut into a very large number of parts and opened as shown in the figure, the circumference of the circle becomes, practically, a straight line.

Note.—Imagine the circle cut into an infinite number of parts and thus opened and the circumference to be a straight line.

Observe that a circle may be regarded as made up of an infinite number of triangles whose united bases equal the circumference and whose altitude equals the radius. Hence to find the area of a circle we have the following:

Rule I. *Multiply the circumference by $\frac{1}{4}$ of the diameter.*

2. It has already been stated that if the diameter of a circle is 1, its circumference is 3.141592.* Hence the area of a circle whose diameter is 1 is $(3.141592 \times \frac{1}{4})$ .785398.

3. A circle whose diameter is 2, is 4 times as large as a circle whose diameter is 1; a circle whose diameter is 3, 9 times as large, etc. Hence to find the area of a circle we have also the following:

Rule II. *Multiply the square of the diameter by .785398.*

4. The approximate area may be found by taking $\frac{78}{100}$ (or .78) of the square of the diameter. See Werner Arithmetic, Book II., page 256.

* See page 229, note.

339. Miscellaneous Problems.*

1. Find the approximate area of a circle whose diameter is 20 feet.

2. What is the area of a circle whose diameter is 1 foot? 1 yard? 1 rod? 1 mile?

3. What is the area of a circle whose diameter is 2 feet? 2 yards? 2 rods? 2 miles?

4. A horse is so fastened with a rope halter that he can feed over a circle 40 feet in diameter. Does he feed over more or less than 5 square rods?

5. Find the approximate length (in rods) of the side of a square containing 2 acres.

6. Find the approximate diameter (in rods) of a circle whose circumference is one mile.

7. Find the approximate area of the circle described in problem 6.

8. Find the approximate circumference of a circle whose diameter is 30 rods.

9. The expression "a bicycle geared to 68" means that the machine is so geared that it will move forward at each revolution of the pedal shaft as far as a 68-inch wheel would move forward at one revolution. How far does a bicycle "geared to 68" move forward at each revolution of the pedal shaft? A bicycle "geared to 70"?

10. What is the approximate circumference of the largest circle that can be drawn on the floor of a room 40 ft. by 40 ft. if at its nearest points the circumference is 2 feet from the edge of the floor?

*Require the pupil to make an estimate of the answer to each problem before attempting to solve it with the aid of a pencil.

DENOMINATE NUMBERS.

VOLUME MEASURE.

340. The standard unit of volume measure is a **cubic yard** which is the equivalent of a 1-yard cube. This unit, like the cubic foot and the cubic inch, is derived from the corresponding unit of linear measure.

CUBIC MEASURE.

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).

EXERCISE.

1. Show by a drawing that there are 27 cu. ft. in a 1-yard cube.
2. How many cubic inches in 1 half of a cubic foot?
3. How many cubic inches in a $\frac{1}{2}$-foot cube?
4. How many cubic feet in 1 third of a cubic yard?
5. How many cubic feet in a $\frac{1}{3}$-yard cube?
6. Estimate in cubic feet the amount of air in the school room.
7. Estimate in cubic yards the amount of air in the school room.
8. Estimate in cubic inches the capacity of your dinner box.
9. Estimate in cubic feet the capacity of some wagon box.
10. Estimate in cubic inches the volume of the school globe.*

* A globe is a little more than $\frac{1}{2}$ of the smallest cube from which it could have been made. See Werner Arithmetic, Book II., p. 266.

Denominate Numbers—Volume Measure.

341. Wood is usually measured by the **cord.** A cord is a pile 4 feet wide, 4 feet high, and 8 feet long or its equivalent. Hence—

128 cubic feet = 1 cord.

PROBLEMS.

1. Estimate the number of cords of wood that could be put upon the floor of the school room if the desks were removed and the wood piled to the depth of four feet.

2. If 4-foot wood is piled 6 feet high what must be the length of the pile to contain 100 cords?

3. How many cords of wood in a pile 8 feet wide, 8 feet high, and 16 feet long?

4. Compare the amount of wood in the pile described in problem 3, with the amount in a pile one half as wide, one half as high, and one half as long.

5. If I pay $1.10 a cord for sawing wood, cutting each 4-foot stick into 3 pieces, how much ought I to pay for cutting each 4-foot stick into 4 pieces?

6. A pile of wood 4 ft. high, 4 ft. wide, and 192 ft. long contains ——— cords. How many cords in a pile 4 feet high, 192 feet long, and 46 inches wide?

7. A pile of wood is as wide as it is high and 32 feet long. It contains 9 cords. What is the width and height of the pile?

8. How many cords of 4-foot wood can be piled in a cellar that is 24 feet wide and 32 feet long, provided the pile is 4 feet high and one end of each 4-foot stick touches a wall of the cellar?

9. How many cords of wood in a pile that is 16 inches wide, 4 feet high, and 32 feet long?

Denominate Numbers—Volume Measure.

342. Rough stone is usually measured by the **cord**. A pile 4 feet high, 4 feet wide, and 8 feet long or its equivalent, is 1 cord.

NOTE.—One cord of good stone is sufficient for about 100 cubic feet of wall. Hence in estimates it is customary to use the number 100 instead of 128; that is, as many cords of stone will be required for a given wall as 100 cubic feet is contained times in the number of cubic feet in the wall.

PROBLEMS.

1. Estimate the number of cords of stone necessary for a cellar wall 18 inches thick, the inside dimensions of the cellar being 15 feet by 18 feet and 7 feet deep, no allowance being made for openings in the wall.

2. What are the outside dimensions of the wall of the cellar described in problem 1?

3. What length of wall 7 feet high and 18 inches thick is equivalent, so far as amount of stone is concerned, to the cellar wall described in problem 1?

4. If $\frac{3}{4}$ of the depth of the cellar described above is to be below the surface of the ground, how many cubic yards of earth must be excavated?

5. How many per cent less of stone will be required for a 16-inch wall than for an 18-inch wall?

6. Estimate the stone necessary for a wall 100 yards long, 11 feet high, and 2 feet thick.

7. If the specific gravity of stone is $2\frac{1}{2}$ and each cord is equivalent to 100 solid feet, how much does a cord of stone weigh?

8. If the specific gravity of a certain stone is $2\frac{1}{4}$, what is the weight of a block 8 feet by 2 feet by 2 feet?

Denominate Numbers—Volume Measure.

343. An ordinary **brick** is 2 in. by 4 in. by 8 in. and weighs about 4 pounds.

PROBLEMS.

1. How many bricks are equivalent to 1 cubic foot?

NOTE.—When bricks are laid in mortar in the usual way, about 22 bricks are required to make a cubic foot of wall.

2. Estimate the number of bricks necessary for a cellar wall 12 inches thick, the inside dimensions of the cellar being 15 feet by 18 feet, and 7 feet deep, no allowance being made for openings in the wall?

3. What are the outside dimensions of the wall of the cellar described in problem 2?

4. What length of wall 7 feet high and 12 inches thick is equivalent, so far as the number of bricks required is concerned, to the cellar wall described in problem 2?

5. If $\frac{3}{7}$ of the depth of the cellar described above is to be below the surface, how many cubic yards of earth must be excavated?

6. Estimate the number of bricks necessary for a wall 100 yards long, 11 feet high, and 1 foot thick.

7. If a brick is exactly 2 in. by 4 in. by 8 in. and weighs exactly $4\frac{1}{4}$ lbs. what is its specific gravity?*

8. Find the approximate weight (in tons) of a pile of bricks as long as your school-room, 2 feet wide, and 4 feet high.

9. Find the approximate weight of a chimney, outside dimensions, 16 in. by 16 in., and 20 ft. high, the flue being 8 in. by 8 in.

*Compare the weight of 27 such bricks with the weight of a cubic foot of water.

Denominate Numbers—Lumber.

344. A **foot of lumber** is a board 1 foot square and 1 inch thick or its equivalent.

NOTE 1.—An exception to the foregoing is made in the measurement of boards less than 1 inch in thickness. A square foot of such boards is regarded as a foot of lumber whatever the thickness.

Observe that a piece of board 1 inch wide, 1 inch thick, and 12 feet long, is 1 foot of lumber.

EXERCISE.

Tell the number of feet of lumber in each of the following boards, the thickness in each case being 1 inch (or less):

1 in. wide and 12 ft. long. 2 in. wide and 12 ft. long.
3 in. wide and 12 ft. long. 4 in. wide and 12 ft. long.
7 in. wide and 12 ft. long. 13 in. wide and 12 ft. long.
9 in. wide and 12 ft. long. 12 in. wide and 12 ft. long.

(a) How many feet (of lumber) in the eight boards?

PROBLEMS.

1. How much lumber in 6, 12-ft., 1-in. boards whose widths are 11 in., 13 in., 9 in., 10 in., 12 in., and 14 in.?

2. How much lumber in 5, 12-ft. $\frac{3}{4}$-in. boards whose widths are 10 in., 12 in., 12 in., 11 in., and 14 in.?

3. How much lumber in 7, 12-ft. $\frac{1}{2}$-in. boards whose widths are 9 in., 8 in., 5 in., 7 in., 8 in., 6 in., and 9 in.?

4. How much lumber in 8, 12-ft., 1-in. boards each of which is 12 inches wide?

5. How much lumber in 54, 12-ft. 1-in. boards each of which is 6 inches wide?

(b) Find the sum of the five results.

Denominate Numbers—Lumber.

PROBLEMS.

NOTE 2.—**A 14-foot board** contains $\frac{1}{8}$ more lumber than a 12-ft. board of the same width and thickness. Hence to find the number of feet of lumber in 14-foot boards, find the number of feet in as many 12-foot boards* and add to the result $\frac{1}{8}$ of itself.

1. How much lumber in 6, 14-ft., 1-in. boards whose widths are 11 in., 12 in., 12 in., 15 in., 10 in., and 13 in.?
2. How much lumber in a pile of 14-ft. boards whose united width is 8 feet 7 inches?
3. How much lumber in 56, 14-ft. boards each of which is 6 inches wide?†
4. How much lumber in 24, 14-ft. boards each of which is 12 inches wide?‡

(a) Find the sum of the four results.

PROBLEMS.

NOTE 3.—**A 16-foot board** contains $\frac{1}{3}$ more lumber than a 12-foot board of the same width and thickness. Make a rule for finding the number of feet of lumber in 16-foot boards.

1. How much lumber in 6, 16-ft. 1-in. boards whose widths are 12 in., 10 in , 14 in., 13 in., 12 in., and 10 in.?
2. How much lumber in a pile of 16-foot boards whose united width is 9 feet 8 inches?
3. How much lumber in 48, 16-ft. boards each of which is 6 inches wide?
4. How much lumber in 34, 16-ft. boards each of which is 12 inches wide?

(b) Find the sum of the four results.

* Take the nearest integral number of feet.
† How much lumber in 1 14-foot board 6 inches wide?
‡ How much lumber in 1 14-foot board 12 inches wide?

Denominate Numbers—Lumber.

PROBLEMS.

NOTE 4.—**A 1¼-inch board** contains ¼ more lumber than a 1-inch board of the same width and length. A 1½-inch board contains ½ more lumber than a 1-inch board of the same width and length.

1. How much lumber in 4, 12-foot, 1¼-in. boards whose widths are 12 in., 13 in., 14 in., and 13 in.?

2. How much lumber in 4, 16-foot, 1½-in. boards whose widths are 13 in., 16 in., 12 in., and 13 in.?

3. How much lumber in 4, 18-foot, 1½-in. boards, each of which is 12 inches wide?

4. How much lumber in 4, 16-ft., 1½-in. boards, each of which is 6 inches wide?

(a) Find the sum of the four results.

PROBLEMS.

NOTE 5.—**A "2 by 4, 12"** is a piece of lumber 2 in. thick, 4 in. wide, and 12 feet long.

Find the number of feet of lumber in each of the following items:

1. 16 pieces 2 × 4, 12.
2. 18 pieces 4 × 4, 12.
3. 25 pieces 2 × 8, 12.
4. 30 pieces 2 × 6, 12.
5. 20 pieces 4 × 6, 12.
6. 32 pieces 6 × 6, 12.

(b) Find the sum of the six results.

Observe that *in a 12-foot piece of lumber* there are as many feet as there are square inches in the cross-section. A piece of lumber 1 in. by 1 in. and 12 feet long is 1 foot of lumber; a piece 2 in. by 2 in. is 4 feet of lumber; a piece 2 in. by 3 in. is 6 feet of lumber, etc.

Denominate Numbers—Lumber.

PROBLEMS.

NOTE 6.—In the measurement of **timbers** of all sizes it is customary to consider each piece as containing the integral number of feet nearest to the actual content. Thus, a piece of 2 × 4, 14, actually contains 9⅓ feet, but in all lumber yards it is counted as 9 feet. A piece of 2 × 4, 16, actually contains 10⅔ feet, but it is counted as 11 feet.

Find the number of feet of lumber in each of the following items:

1. 16 pieces 2 × 4, 14.*
2. 24 pieces 4 × 4, 14.
3. 32 pieces 2 × 8, 14.
4. 17 pieces 4 × 6, 14.
5. 15 pieces 8 × 8, 16.
6. 12 pieces 4 × 10, 16.
7. 14 pieces 8 × 12, 16.
8. 6 pieces 12 × 12, 24.

(a) Find the sum of the eight results.

PROBLEMS.

NOTE 7.—"Lumber at $15 per M," means that the lumber is sold at the rate of $15 per 1000 feet.

Find the cost:

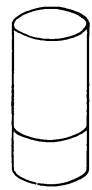
1. 26, 16-foot, 6-in. fence boards @ $15 per M.
2. 34, 14-foot, 12-in. stock boards " $18 per M.
3. 20 pieces 2 × 4, 16, " $16 per M.
4. 14 pieces 4 × 6, 18, " $16 per M.
5. 25 pieces 4 × 6, 16, " $15 per M.
6. 18 pieces 4 × 4, 14, " $15 per M.

(b) Find the sum of the six results.

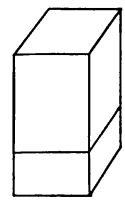
* Find the number of feet in 1 piece of 2 × 4, 14 (nearest whole number of feet) and multiply by 16.

Geometry.

345. To find the solid content of a **cylinder** or of a **right prism.***



Observe that in any cylinder or right prism the number of cubic units in one layer 1 unit high (as indicated in the diagrams) is equal to the number of square units in the area of the base. Thus, if there are $4\frac{1}{2}$ square units in the area of the base there are $4\frac{1}{2}$ cubic units in one layer. The content of the entire solid is as many times the cubic units in one layer, as the solid is linear units in height. Hence the rule as usually given: "*Multiply the area of the base by the altitude.*"



To THE TEACHER.—This rule must be carefully interpreted by the pupil. He must not be allowed the misconception that area multiplied by any number can give solid content, except *through such interpretation as is suggested in the above observation.*

PROBLEMS.

1. Find the solid content of a square right prism whose base is 6 in. by 6 in., and whose altitude is 8 inches.

2. Find the approximate solid content of a cylinder 6 inches in diameter and 10 inches long.

* A right prism is a solid whose bases, or ends, are similar, equal, and parallel plane polygons, and whose lateral faces are perpendicular to its bases.

346. Miscellaneous Problems.

1. Find the solid content of an octagonal right prism the area of whose base is 24 square inches and whose altitude is 15 inches.

2. What is the solid content of a cylinder or of any right prism, the area of whose base is 30 square inches and whose altitude is 12 inches?

3. How many cubic feet of earth must be removed to dig a well 6 feet in diameter and 20 feet deep? *

4. Find the approximate number of feet of $1\frac{1}{2}$-in. lumber required to make the lining of the sides of a cylindrical silo that is 20 feet in diameter and 30 feet deep.

5. Find the approximate number of cords of rough stone in a cylindrical pile that is 16 feet in diameter and six feet deep.

6. Find the approximate number of brick necessary for a solid cylindrical foundation that is 9 feet in diameter and 4 feet high.

7. If the average specific gravity of the brick and mortar used in the foundation described in problem 6, is 1.9, how much does the entire foundation weigh?

8. Find the weight in kilograms of a column of water 1 decimeter square and 10 meters deep.

9. Find the weight in pounds of 1000 feet of white pine 1-inch boards, the specific gravity being .6.

10. Find the weight of a load (1 cubic yard) of wet sand, the specific gravity being exactly 2.

* The exact number of cubic feet cannot be expressed in figures. An approximation that will answer many practical purposes may be obtained by regarding the circle (base) as $\frac{3}{4}$ of its circumscribed square. If an answer more nearly accurate is required use .78 instead of $\frac{3}{4}$.

DENOMINATE NUMBERS.

Capacity.

347. The standard unit of capacity used in measuring liquids is a **gallon.** *A gallon equals 231 cubic inches.*

Liquid Measure.

4 gills (gi.)	= 1 pint (pt.).
2 pints	= 1 quart (qt.).
4 quarts	= 1 gallon (gal.).
31½ gallons	= 1 barrel (bbl.).

Observe that 1 cubic foot = nearly 7½ gallons.

Observe that 4.2 cubic feet = nearly 1 barrel.

A kerosene barrel contains about 52 gallons. It equals nearly 7 cubic feet.

PROBLEMS.

1. Find the capacity (approximate or exact), in gallons, of a rectangular tank 3 ft. by 4 ft. by 8 ft.

2. Find the approximate capacity, in gallons, of a cylindrical tank 4 feet in diameter and 4 feet deep.

3. Find the approximate capacity, in barrels (31½ gal.), of a rectangular tank 2 ft. by 4 ft. by 12 ft.

4. Find the approximate capacity, in barrels (31½ gal.), of a cylindrical cistern 6 ft. in diameter and 6 ft. deep.

5. Find the approximate capacity, in barrels (31½ gal.), of a cylindrical cistern 12 ft. in diameter and 6 ft. deep.

6. Find the approximate capacity, in barrels (31½ gal.), of a cylindrical cistern 12 ft. in diameter and 12 ft. deep.

Denominate Numbers—Capacity.

348. The standard unit of capacity used in measuring grain, fruits, vegetables, lime, coal, etc., is a **bushel.** *A bushel equals 2150.4 cubic inches.*

NOTE.—In measuring large fruits, vegetables, lime, and coal, the unit is the "**heaped bushel.**" *A heaped bushel equals about* 1¼ "*stricken bushels.*"

DRY MEASURE.

2 pints (pt.) = 1 quart (qt.).
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).

A bushel is nearly 1¼ cubic feet.
A "heaped bushel" is about 1½ cubic feet.
A "dry gallon" (4 quarts dry measure) equals 268.8 cubic inches.

Enough "**ear corn**" to make, when shelled, one bushel, occupies about 2¼ cubic feet. If the corn is inferior in quality it will occupy more space than this—sometimes 2½ cubic feet.

PROBLEMS.

1. Find the capacity in bushels of a wheat bin 8 ft. by 8 ft. by 10 ft.*
2. Give the dimensions of the smallest bin in which 1000 bushels of oats may be stored.
3. In a bin 12 feet square, there is rye to the depth of 7½ feet. How many bushels?
4. How many bushels of potatoes (without heaping the bin) may be stored in a bin that is 8 ft. by 4 ft. by 6 ft.?
5. If the corn is of excellent quality, how many bushels of "shelled corn" may be expected from a crib of ear corn, 8 ft. by 10 ft. by 80 ft.?

*For many practical purposes the approximate ratio (1½) of the bushel to the cubic foot will give in such problems as these, results sufficiently accurate.

Denominate Numbers—Weight.

349. The standard unit of weight in common use is a **pound Avoirdupois.**

AVOIRDUPOIS WEIGHT.

16 ounces (oz.) = 1 pound (lb.).
2000 pounds = 1 ton (T.).

The abbreviation for 1 hundredweight (100 lb.) is *cwt.*

MISCELLANEOUS WEIGHTS.

1 gallon of water	= about 8⅓ lb.
1 gallon of milk	= about 8 6 lb.
1 gallon of kerosene	= about 6½ lb.
1 cubic foot of water	= 62½ lb.
1 bushel of wheat	= 60 lb.
1 bushel of beans	= 60 lb.
1 bushel of clover seed	= 60 lb.
1 bushel of potatoes	= 60 lb.
1 bushel of shelled corn	= 56 lb.
1 bushel of ear corn	= 70 lb.*
1 bushel of rye	= 56 lb.
1 bushel of barley	= 48 lb.
1 bushel of oats	= 32 lb.
1 barrel of flour	= 196 lb.
1 barrel of beef or pork	= 200 lb.

PROBLEMS.

Find the cost—

1. Of 2650 lb. coal at $5.50 per ton.
2. Of 2650 lb. oats at 24¢ a bushel.
3. Of 3330 lb. wheat at 80¢ a bushel.
4. Of 4650 lb. potatoes at 42¢ a bushel.

(a) Find the sum of the four results.

*This means the amount of ear corn required to make 1 bushel of shelled corn.

Denominate Numbers—Weight.

PROBLEMS.

Find the cost—

1. Of 2560 lb. hay at $7.50 per ton.
2. Of 1430 lb. straw at 30¢ per cwt.
3. Of $2\frac{1}{4}$ tons meal at $\frac{3}{4}$ of a cent a pound.
4. Of $1\frac{1}{2}$ tons corn husks at $1\frac{1}{4}$ cents a pound.
5. Of 3420 lb. hay at $8.00 per ton.

(a) Find the sum of the five results.

Find the cost—

6. Of 2140 lb. oats at 24¢ a bushel.
7. Of 2140 lb. corn at 28¢ a bushel.
8. Of 2140 lb. wheat at 90¢ a bushel.
9. Of 2140 lb. barley at 36¢ a bushel.
10. Of 2140 lb. rye at 42¢ a bushel.

(b) Find the sum of the five results.

Find the cost—

11. Of 520 lb. clover seed at $6.30 a bushel.
12. Of 520 lb. potatoes at 75¢ a bushel.
13. Of 520 lb. beans at $2.15 a bushel.
14. Of 520 lb. corn at 20¢ a bushel.
15. Of 520 lb. ear corn at 35¢ a bushel.

(c) Find the sum of the five results.

Find the approximate weight—

16. Of a barrel of kerosene.
17. Of 1 quart of milk.

18. Of the oats that will fill a bin that is 4 ft. by 4 ft by 9 ft.

19. Of the water that will fill a tank that is 2 ft. by 2 ft. by 12 ft.

20. Of a five-gallon can of kerosene.

Denominate Numbers—Weight.

350. Troy weight is used in weighing gold, silver, and jewels.

TROY WEIGHT.

24 grains (gr.) = 1 pennyweight (pwt.).
20 pennyweights = 1 ounce (oz.).
12 ounces = 1 pound (lb.).

351. Apothecaries' weight is used in mixing medicines and in selling them at retail.

APOTHECARIES' WEIGHT.

20 grains (gr.) = 1 scruple (℈).
3 scruples = 1 dram (ʒ).
8 drams = 1 ounce (℥).
12 ounces = 1 pound (℔).

NOTE 1.—The pound Troy and the pound Apothecary are equal, each weighing 5760 grains. The pound Avoirdupois weighs 7000 Troy or Apothecary grains.

NOTE 2.—The ounce Troy and the ounce Apothecary are each 480 grains; the ounce Avoirdupois is 437½ grains.

QUERY.—Which is heavier, a pound of feathers or a pound of gold? An ounce of feathers or an ounce of gold?

PROBLEMS

1. Change 5 lb. Avoirdupois weight to pounds, ounces, etc., Apothecaries' weight.

2. How many 5 gr. powders can be made from one Avoirdupois ounce of quinine?

3. One Avoirdupois ton of gold is how many Troy pounds?

4. Twenty-four Troy pounds equal how many Avoirdupois pounds?

352. Miscellaneous Tables.

12 units = 1 dozen.	20 units = 1 score.
12 dozen = 1 gross.	24 sheets (paper) = 1 quire.
12 gross = 1 great gross.	20 quires = 1 ream.

353. Apothecaries' Fluid Measure.

60 minims (m) = 1 fluid dram (ƒ 3).
8 fluid drams = 1 fluid ounce (ƒ ℥).
16 fluid ounces = 1 pint (O).
8 pints = 1 gallon (Cong.).
One minim of water is about 1 drop.

354. Angular Measure.

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
360 degrees = 1 circumference.

355. Measure of Time.

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
365 days = 1 common year.
52 wk. 1 da. = 1 common year.
366 days = 1 leap year.
52 wk. 2 da. = 1 leap year.

"*So teach us to number our days that we may apply our hearts unto wisdom.*"

SHORT METHODS

IN

MULTIPLICATION AND DIVISION.

Art. 1. To multiply a number by 50: *Multiply* $\frac{1}{2}$ *of the number by* 100. Why?

46×50	$\frac{1}{2}$ of $46 \times 100 = 2300$
47×50	$\frac{1}{2}$ of $47 \times 100 = 2350$

Multiply:

44 by 50	32 by 50	35 by 50
36 by 50	38 by 50	27 by 50
64 by 50	42 by 50	55 by 50
82 by 50	76 by 50	43 by 50

(a) Find the sum of the twelve products.

Art. 2. To multiply a number by 51: *Take* 50 *times the number, to which add the number itself.* Why?

48×51	50 times $48 = 2400$	$2400 + 48 = 2448$
37×51	50 times $37 = 1850$	$1850 + 37 = 1887$

Multiply:

26 by 51	34 by 51	35 by 51
46 by 51	32 by 51	29 by 51
24 by 51	42 by 51	43 by 51
66 by 51	84 by 51	33 by 51

(b) Find the sum of the twelve products.

Art. 3. To multiply a number by 52: *Take* 50 *times the number, to which add twice the number.* Why?

34×52	50 times 34 = 1700	1700 + 68 = 1768
45×52	50 times 45 = 2250	2250 + 90 = 2340

Multiply:

26 by 52	38 by 52	27 by 52
18 by 52	14 by 52	17 by 52
24 by 52	32 by 52	35 by 52
36 by 52	44 by 52	23 by 52

(c) Find the sum of the twelve products.

Art. 4. To multiply a number by 49: *Take* 50 *times the number, from which subtract the number itself.* Why?

24×49	50 times 24 = 1200	1200 − 24 = 1176
33×49	50 times 33 = 1650	1650 − 33 = 1617

Multiply:

18 by 49	22 by 49	27 by 49
28 by 49	34 by 49	35 by 49
16 by 49	46 by 49	43 by 49
26 by 49	38 by 49	31 by 49

(d) Find the sum of the twelve products.

Art. 5. To multiply a number by $33\frac{1}{3}$: *Multiply* $\frac{1}{3}$ *of the number by* 100. Why?

$36 \times 33\frac{1}{3}$	$\frac{1}{3}$ of $36 \times 100 = 1200$
$37 \times 33\frac{1}{3}$	$\frac{1}{3}$ of $37 \times 100 = 1233\frac{1}{3}$

Multiply:

24 by $33\frac{1}{3}$	27 by $33\frac{1}{3}$	28 by $33\frac{1}{3}$
18 by $33\frac{1}{3}$	21 by $33\frac{1}{3}$	22 by $33\frac{1}{3}$
30 by $33\frac{1}{3}$	33 by $33\frac{1}{3}$	35 by $33\frac{1}{3}$
15 by $33\frac{1}{3}$	39 by $33\frac{1}{3}$	41 by $33\frac{1}{3}$

(e) Find the sum of the twelve products.

Art. 6. To multiply a number by $34\frac{1}{3}$: *Take* $33\frac{1}{3}$ *times the number, to which add the number itself.* Why?

$24 \times 34\frac{1}{3}$ $33\frac{1}{3}$ times $24 = 800$ $800 + 24 = 824$
$25 \times 34\frac{1}{3}$ $33\frac{1}{3}$ times $25 = 833\frac{1}{3}$ $833\frac{1}{3} + 25 = 858\frac{1}{3}$

Multiply:

18 by $34\frac{1}{3}$	27 by $34\frac{1}{3}$	24 by $34\frac{1}{3}$
21 by $34\frac{1}{3}$	33 by $34\frac{1}{3}$	39 by $34\frac{1}{3}$
30 by $34\frac{1}{3}$	36 by $34\frac{1}{3}$	15 by $34\frac{1}{3}$
12 by $34\frac{1}{3}$	16 by $34\frac{1}{3}$	17 by $34\frac{1}{3}$

(f) Find the sum of the twelve products.

Art. 7. To multiply a number by $35\frac{1}{3}$: *Take* $33\frac{1}{3}$ *times the number, to which add twice the number.* Why?

$24 \times 35\frac{1}{3}$ $33\frac{1}{3}$ times $24 = 800$ $800 + 48 = 848$
$25 \times 35\frac{1}{3}$ $33\frac{1}{3}$ times $25 = 833\frac{1}{3}$ $833\frac{1}{3} + 50 = 883\frac{1}{3}$

Multiply:

21 by $35\frac{1}{3}$	30 by $35\frac{1}{3}$	27 by $35\frac{1}{3}$
18 by $35\frac{1}{3}$	66 by $35\frac{1}{3}$	36 by $35\frac{1}{3}$
33 by $35\frac{1}{3}$	39 by $35\frac{1}{3}$	31 by $35\frac{1}{3}$
15 by $35\frac{1}{3}$	20 by $35\frac{1}{3}$	42 by $35\frac{1}{3}$

(g) Find the sum of the twelve products.

Art. 8. To multiply a number by $32\frac{1}{3}$: *Take* $33\frac{1}{3}$ *times the number, from which subtract the number itself.* Why?

$21 \times 32\frac{1}{3}$ $33\frac{1}{3}$ times $21 = 700$ $700 - 21 = 679$
$22 \times 32\frac{1}{3}$ $33\frac{1}{3}$ times $22 = 733\frac{1}{3}$ $733\frac{1}{3} - 22 = 711\frac{1}{3}$

Multiply:

15 by $32\frac{1}{3}$	21 by $32\frac{1}{3}$	18 by $32\frac{1}{3}$
24 by $32\frac{1}{3}$	30 by $32\frac{1}{3}$	27 by $32\frac{1}{3}$
33 by $32\frac{1}{3}$	39 by $32\frac{1}{3}$	36 by $32\frac{1}{3}$
66 by $32\frac{1}{3}$	19 by $32\frac{1}{3}$	20 by $32\frac{1}{3}$

(h) Find the sum of the twelve products.

Art. 9. To multiply a number by 25: *Multiply* $\frac{1}{4}$ *of the number by* 100. Why?

48×25	$\frac{1}{4}$ of $48 \times 100 = 1200$
49×25	$\frac{1}{4}$ of $49 \times 100 = 1225$

Multiply:

36 by 25	32 by 25	33 by 25
40 by 25	28 by 25	29 by 25
24 by 25	16 by 25	19 by 25
52 by 25	48 by 25	50 by 25

(i) Find the sum of the twelve products.

Art. 10. To multiply a number by 26: *Take* 25 *times the number, to which add the number itself.* Why?

36×26	25 times $36 = 900$	$900 + 36 = 936$
37×26	25 times $37 = 925$	$925 + 37 = 962$

Multiply:

36 by 26	48 by 26	45 by 26
28 by 26	24 by 26	25 by 26
44 by 26	32 by 26	35 by 26
52 by 26	16 by 26	18 by 26

(j) Find the sum of the twelve products.

Art. 11. To multiply a number by 27: *Take* 25 *times the number, to which add twice the number.* Why?

36×27	25 times $36 = 900$	$900 + 72 = 972$
37×27	25 times $37 = 925$	$925 + 74 = 999$

Multiply:

48 by 27	52 by 27	37 by 27
32 by 27	16 by 27	17 by 27
28 by 27	24 by 27	26 by 27
44 by 27	56 by 27	35 by 27

(k) Find the sum of the twelve products.

Art. 12. To multiply a number by 24: *Take* 25 *times the number, from which subtract the number itself.* Why?

32×24	25 times 32 = 800	$800 - 32 = 768$
33×24	25 times 33 = 825	$825 - 33 = 792$

Multiply:

24 by 24	16 by 24	17 by 24
44 by 24	36 by 24	37 by 24
28 by 24	32 by 24	35 by 24
48 by 24	52 by 24	54 by 24

(l) Find the sum of the twelve products.

Art. 13. To multiply a number by 20: *Multiply* $\frac{1}{5}$ *of the number by* 100. How may a number be multiplied by 21? by 22? by 19?

35×21	20 times 35 = 700	$700 + 35 = 735$

Multiply:

45 by 21	45 by 22	45 by 19
35 by 21	35 by 22	35 by 19
36 by 21	36 by 22	36 by 19
27 by 21	27 by 22	27 by 19

(m) Find the sum of the twelve products.

Art. 14. To multiply a number by $16\frac{2}{3}$: *Multiply* $\frac{1}{6}$ *of the number by* 100. How may a number be multiplied by $17\frac{2}{3}$? by $18\frac{2}{3}$? by $15\frac{2}{3}$?

$24 \times 17\frac{2}{3}$	$16\frac{2}{3}$ times 24 = 400	$400 + 24 = 424$

Multiply:

18 by $17\frac{2}{3}$	18 by $18\frac{2}{3}$	18 by $15\frac{2}{3}$
30 by $17\frac{2}{3}$	30 by $18\frac{2}{3}$	30 by $15\frac{2}{3}$
36 by $17\frac{2}{3}$	36 by $18\frac{2}{3}$	36 by $15\frac{2}{3}$
42 by $17\frac{2}{3}$	42 by $18\frac{2}{3}$	42 by $15\frac{2}{3}$

(n) Find the sum of the twelve products.

Art. 15. To multiply a number by $12\frac{1}{2}$: *Multiply $\frac{1}{8}$ of the number by* 100. How may a number be multiplied by $13\frac{1}{2}$? by $14\frac{1}{2}$? by $11\frac{1}{2}$?

$32 \times 13\frac{1}{2}$ $12\frac{1}{2}$ times 32 = 400 400 + 32 = 432

Multiply:

24 by $13\frac{1}{2}$	24 by $14\frac{1}{2}$	24 by $11\frac{1}{2}$
16 by $13\frac{1}{2}$	16 by $14\frac{1}{2}$	16 by $11\frac{1}{2}$
40 by $13\frac{1}{2}$	40 by $14\frac{1}{2}$	40 by $11\frac{1}{2}$
48 by $13\frac{1}{2}$	48 by $14\frac{1}{2}$	48 by $11\frac{1}{2}$

(o) Find the sum of the twelve products.

Art. 16. To multiply a number by 125: *Multiply $\frac{1}{8}$ of the number by* 1000. How may a number be multiplied by 126? by 127? by 124?

96×125 $\frac{1}{8}$ of $96 \times 1000 = 12000$

96×126 125 times 96 = 12000 12000 + 96 = 12096

Multiply:

120 by 126	120 by 127	120 by 124
320 by 126	320 by 127	320 by 124
240 by 126	240 by 127	240 by 124

(p) Find the sum of the nine products.

Art. 17. To multiply a number by 250: *Multiply $\frac{1}{4}$ of the number by* 1000. How may a number be multiplied by 251? by 252? by 249?

48×250 $\frac{1}{4}$ of $48 \times 1000 = 12000$

48×251 250 times 48 = 12000 12000 + 48 = 12048

Multiply:

60 by 251	60 by 252	60 by 249
72 by 251	72 by 252	72 by 249
96 by 251	96 by 252	96 by 249

(q) Find the sum of the nine products.

Art. 18. To square $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, etc.: *Multiply the integer by the integer plus* 1, *and add* $\frac{1}{4}$ *to the product.*

$2\frac{1}{2} \times 2\frac{1}{2} = \overline{2 \text{ times } 2} + \overline{2 \text{ times } \frac{1}{2}} + \overline{\frac{1}{2} \text{ of } 2} + \overline{\frac{1}{2} \text{ of } \frac{1}{2}}$

But $\overline{2 \text{ times } \frac{1}{2}} + \frac{1}{2}$ of 2 = 1 time 2; and $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$

Hence, $2\frac{1}{2} \times 2\frac{1}{2} = \overline{2 \times 3} + \frac{1}{4} = 6\frac{1}{4}$

$3\frac{1}{2} \times 3\frac{1}{2} = \overline{3 \times 4} + \frac{1}{4} = 12\frac{1}{4}$

Multiply:

$4\frac{1}{2}$ by $4\frac{1}{2}$	$5\frac{1}{2}$ by $5\frac{1}{2}$	$6\frac{1}{2}$ by $6\frac{1}{2}$
$7\frac{1}{2}$ by $7\frac{1}{2}$	$8\frac{1}{2}$ by $8\frac{1}{2}$	$9\frac{1}{2}$ by $9\frac{1}{2}$
$1\frac{1}{2}$ by $1\frac{1}{2}$	$2\frac{1}{2}$ by $2\frac{1}{2}$	$3\frac{1}{2}$ by $3\frac{1}{2}$

(r) Find the sum of the nine products.

Art. 19. To square 25, 35, 45, etc.: *Multiply the tens' figure* by the tens' figure increased by* 1; *regard the product as hundreds, to which add* 25.

To explain this rule, think of 25 as 2 *tens and* $\frac{1}{2}$ *of a ten*, and apply the explanation given under Art. 18.

$25 \times 25 = \overline{2 \times 3}$ hundred and 25 = 625

$35 \times 35 = \overline{3 \times 4}$ hundred and 25 = 1225

$45 \times 45 = \overline{4 \times 5}$ hundred and 25 = 2025

Multiply:

55 by 55	65 by 65	75 by 75
85 by 85	95 by 95	15 by 15

(s) Find the sum of the six products.

* The author is aware that the expressions "*Multiply the tens' figure*" and "*the tens' figure increased by 1.*" are tabooed by the hypercritical. But it is believed that neither obscurity nor misconception will arise from this use of the word *figure*. The word as here used clearly means the *form value of the figure*—the *number* which the figure by virtue of its shape represents.

Art. 20. To multiply $2\frac{1}{4}$ by $2\frac{3}{4}$, $3\frac{1}{5}$ by $3\frac{4}{5}$, etc.: *Multiply the integer by the integer plus* 1, *and to the product add the product of the fractions.*

Observe that this rule will apply only when the integer of the multiplicand and the integer of the multiplier are the same, and the sum of the fractions is 1.

$2\frac{1}{4}$ by $2\frac{3}{4}$ = $\overline{2 \text{ times } 2} + \overline{2 \text{ times } \frac{1}{4}} + \overline{\frac{3}{4} \text{ of } 2} + \overline{\frac{3}{4} \text{ of } \frac{1}{4}}$

But $\overline{2 \text{ times } \frac{1}{4}} + \overline{\frac{3}{4} \text{ of } 2} = 1$ time 2, and $\overline{\frac{3}{4} \text{ of } \frac{1}{4}} = \frac{3}{16}$

Hence $2\frac{1}{4} \times 2\frac{3}{4} = \overline{2 \times 3} + \overline{\frac{3}{4} \text{ of } \frac{1}{4}} = 6\frac{3}{16}$

$3\frac{1}{5} \times 3\frac{4}{5} = \overline{3 \times 4} + \frac{4}{5} \text{ of } \frac{1}{5} = 12\frac{4}{25}$

$2\frac{1}{3} \times 2\frac{2}{3} = \overline{2 \times 3} + \frac{2}{3} \text{ of } \frac{1}{3} = 6\frac{2}{9}$

Multiply:

$4\frac{1}{3}$ by $4\frac{2}{3}$	$5\frac{1}{4}$ by $5\frac{3}{4}$	$6\frac{1}{5}$ by $6\frac{2}{5}$
$7\frac{1}{4}$ by $7\frac{3}{4}$	$8\frac{2}{3}$ by $8\frac{1}{3}$	$9\frac{1}{4}$ by $9\frac{3}{4}$

(t) Find the sum of the six products.

Art. 21. To multiply 24 by 26, 33 by 37, etc.: *Multiply the tens' figure by the tens' figure increased by* 1; *regard the product as hundreds, to which add the product of the units' figures.*

Observe that this rule will apply only when the tens' figure of the multiplicand and the tens' figure of the multiplier are alike, and the sum of the units' figures is 10.

$22 \times 28 = \overline{2 \times 3}$ hundred and $16 = 616$

$33 \times 37 = \overline{3 \times 4}$ hundred and $21 = 1221$

$44 \times 46 = \overline{4 \times 5}$ hundred and $24 = 2024$

Multiply:

21 by 29	23 by 27	24 by 26
31 by 39	32 by 38	34 by 36
41 by 49	42 by 48	43 by 47

(u) Find the sum of the nine products.

Art. 22. To multiply a number by 15 : *Multiply the number by* 10, *and to the product add* $\frac{1}{2}$ *of the product.*

64×15	10 times 64 = 640	640 + 320 = 960
45×15	10 times 45 = 450	450 + 225 = 675

Multiply :

24 by 15	32 by 15	35 by 15
46 by 15	34 by 15	43 by 15
82 by 15	66 by 15	75 by 15
37 by 15	41 by 15	39 by 15

(v) Find the sum of the twelve products.

Art. 23. To multiply a number by 99 : *Take* 100 *times the number, from which subtract the number itself.* How may a number be multiplied by 98 ?

36×99	100 times 36 = 3600	3600 − 36 = 3564
42×98	100 times 42 = 4200	4200 − 84 = 4116

Multiply :

35 by 99	44 by 99	35 by 98
27 by 99	54 by 99	46 by 98
62 by 99	75 by 99	28 by 98
38 by 99	64 by 99	86 by 98

(w) Find the sum of the twelve products.

Art. 24. To multiply a number by 75 : *Multiply* $\frac{3}{4}$ *of the number by* 100. How may a number be multiplied by $66\frac{2}{3}$? by $62\frac{1}{2}$? by $87\frac{1}{2}$?

Multiply :

64 by 75	24 by $66\frac{2}{3}$	64 by $87\frac{1}{2}$
48 by 75	36 by $66\frac{2}{3}$	48 by $87\frac{1}{2}$
52 by 75	63 by $66\frac{2}{3}$	56 by $87\frac{1}{2}$
37 by 75	37 by $66\frac{2}{3}$	32 by $87\frac{1}{2}$

(x) Find the sum of the twelve products.

Art. 25. To divide a number by 25; by $33\frac{1}{3}$; by $12\frac{1}{2}$; by $16\frac{2}{3}$ by 20; by 50. (See pp. 72, 73, and 74, of this book.)

$$850 \div 25 = \overline{8 \text{ times } 4} + 2 = 34$$

$$933\frac{1}{3} \div 33\frac{1}{3} = \overline{9 \text{ times } 3} + 1 = 28$$

$$637\frac{1}{2} \div 12\frac{1}{2} = \overline{6 \text{ times } 8} + 3 = 51$$

$$750 \div 16\frac{2}{3} = \overline{7 \text{ times } 6} + 3 = 45$$

$$960 \div 20 = \overline{9 \text{ times } 5} + 3 = 48$$

$$450 \div 50 = \overline{4 \text{ times } 2} + 1 = 9$$

Divide:

1275 by 25	$1166\frac{2}{3}$ by $33\frac{1}{3}$	$762\frac{1}{2}$ by $12\frac{1}{2}$
950 by $16\frac{2}{3}$	880 by 20	950 by 50

(y) Find the sum of the six quotients.

NOTE.—Without a pencil, tell the integral quotient and the remainder resulting from the incomplete division of 1584 by 25.

Art. 26. To divide a number by 125; by 250: *Observe that* 125 *is contained in each thousand of a number* 8 *times; that* 250 *is contained in each thousand of a number* 4 *times.*

$$7125 \div 125 = \overline{7 \text{ times } 8} + 1 = 57$$

$$8500 \div 250 = \overline{8 \text{ times } 4} + 2 = 34$$

Divide:

13250 by 250	13500 by 250	13750 by 250
18000 by 250	18750 by 250	18500 by 250
12000 by 125	12500 by 125	12625 by 125
9125 by 125	9375 by 125	9875 by 125

(z) Find the sum of the twelve quotients.

NOTE.—Without a pencil, tell the integral quotient and the remainder resulting from the incomplete division of 15450 by 250.

Art. 27. When the same factor occurs in a dividend and in its divisor, it may be omitted from both without changing their ratio. Hence *all the factors that are common to a dividend and its divisor may be stricken out* (cancelled) *and the quotient* (ratio) *be unchanged.*

NOTE.—In this discussion and in the solution of problems in this connection, all the numbers are regarded as abstract.

I. Divide 180 by 42.

Operation No. 1.

$$\begin{array}{r} 42)180\ (4\tfrac{2}{7} \\ \underline{168} \\ 12 \end{array} \qquad \frac{12}{42} = \frac{2}{7}$$

Operation No. 2.

$$\frac{180}{42} = \frac{\cancel{2} \times 2 \times \cancel{3} \times 3 \times 5}{\cancel{2} \times \cancel{3} \times 7} = \frac{30}{7} = 4\frac{2}{7}$$

Observe that the striking out of the factors 2 and 3 from the dividend and its divisor does not change their ratio—the quotient.

II. Divide 420 by 35.

Operation No. 1.

$$\begin{array}{r} 35)\ 420\ (12 \\ \underline{35} \\ 70 \\ \underline{70} \end{array}$$

Operation No. 2.

$$\frac{420}{35} = \frac{2 \times 2 \times 3 \times \cancel{5} \times \cancel{7}}{\cancel{5} \times \cancel{7}} = \frac{12}{1} = 12$$

Observe that if all the factors of one of the numbers are cancelled the number becomes 1 and not 0. The factor 5 is 5 times 1; the factor 7, 7 times 1. Hence in the above problem there really remain in the divisor, after the cancellation, the factors 1 and $1 = 1 \times 1 = 1$.

III. Divide $48 \times 8 \times 4 = 1536$ by $8 \times 4 \times 4 = 128$.

Operation No. 1.

$$\begin{array}{r} 128)1536\ (12 \\ \underline{128} \\ 256 \\ \underline{256} \end{array}$$

Operation No. 2.

$$\frac{\overset{6}{\cancel{48}} \times \overset{2}{\cancel{8}} \times \cancel{4}}{\cancel{8} \times \cancel{4} \times \cancel{4}} = \frac{12}{1} = 12$$

Observe that it is not necessary to obtain the *prime* factors of a dividend and its divisor to employ cancellation in finding the quotient. In the above the composite factor 8 is stricken out of the divisor and out of the 48 of the dividend.

IV. Divide $56 \times 35 = 1960$ by $15 \times 8 = 120$.

Operation No. 1.

$120)1960\,(16\frac{1}{3}$

120

760

720

40

$\frac{40}{120} = \frac{1}{3}$

Operation No. 2.

$$\frac{\overset{7}{\cancel{56}} \times \overset{7}{\cancel{35}}}{\underset{3}{\cancel{15}} \times \cancel{8}} = \frac{49}{3} = 16\frac{1}{3}$$

Observe that in the above the factor 5 is stricken out of 15 and 35, and the factor 8 is stricken out of the divisor and out of the 56 of the dividend.

MISCELLANEOUS PROBLEMS.

NOTE.—Employ "Short Methods" in the solution of the following problems.

How many cords of wood—

1. In a pile 32 feet by 8 feet by 4 feet?*
2. In a pile 40 feet by 16 feet by 6 feet?
3. In a pile 32 feet by 30 feet by 10 feet?

(aa) Find the sum of the three results.

How many acres of land—

4. In a piece 180 rods by 28 rods?†
5. In a piece 64 rods by 96 rods?
6. In a piece 136 rods by 32 rods?

(bb) Find the sum of the three results.

7. Multiply 64 by 96 and divide the product by $16 \times 24 \times 2$.

8. Multiply 250 by 72 and divide the product by $16\frac{2}{3} \times 3 \times 24$.

(cc) Find the sum of the two results.

*Think of a cord as 8 feet by 4 feet by 4 feet.

†Think of an acre as 40 rods by 4 rods

Find the cost—

9. Of 346 acres of land at $50 per acre.
10. Of 346 acres of land at $51 per acre.
11. Of 346 acres of land at $52 per acre.
12. Of 346 acres of land at $49 per acre.
13. Of 254 acres of land at $51 per acre.

(dd) Find the sum of the five results.

14. Of 243 ft. iron pipe at $33\frac{1}{3}¢$ a foot.
15. Of 243 ft. iron pipe at $34\frac{1}{3}¢$ a foot.
16. Of 243 ft. iron pipe at $35\frac{1}{3}¢$ a foot.
17. Of 243 ft. iron pipe at $32\frac{1}{3}¢$ a foot.
18. Of 156 ft. iron pipe at $35\frac{1}{3}¢$ a foot.

(ee) Find the sum of the five results.

19. Of 260 lb. butter at 25¢ a pound.
20. Of 260 lb. butter at 26¢ a pound.
21. Of 260 lb. butter at 27¢ a pound.
22. Of 260 lb. butter at 24¢ a pound.
23. Of 184 lb. butter at 27¢ a pound.

(ff) Find the sum of the five results.

24. Of 350 lb. coffee at $12\frac{1}{2}¢$ a pound.
25. Of 350 lb. coffee at $13\frac{1}{2}¢$ a pound.
26. Of 350 lb. coffee at $14\frac{1}{2}¢$ a pound.
27. Of 350 lb. coffee at $11\frac{1}{2}¢$ a pound.
28. Of 330 lb. coffee at $16\frac{2}{3}¢$ a pound.
29. Of 330 lb. coffee at $17\frac{2}{3}¢$ a pound.
30. Of 330 lb. coffee at $15\frac{2}{3}¢$ a pound.
31. Of 240 lb. coffee at 25¢ a pound.
32. Of 240 lb. coffee at 26¢ a pound.
33. Of 240 lb. coffee at 27¢ a pound.

(gg) Find the sum of the ten results.

Find the cost—

34. Of $2\frac{1}{2}$ tons coal at \$$2\frac{1}{2}$ per ton.
35. Of $3\frac{1}{2}$ tons coal at \$$3\frac{1}{2}$ per ton.
36. Of $4\frac{1}{2}$ tons coal at \$$4\frac{1}{2}$ per ton.

(hh) Find the sum of the three results.

37. Of 25 tons of meal at \$25 per ton.
38. Of 35 acres of land at \$35 per acre.
39. Of 45 M. ft. of lumber at \$45 per M.

(ii) Find the sum of the three results.

40. Of 23 yd. cloth at 27¢ a yard.
41. Of 36 yd. cloth at 34¢ a yard.
42. Of 42 yd. cloth at 48¢ a yard.

(jj) Find the sum of the three results.

43. Of 3240 ft. lumber at \$15 per M.
44. Of 2460 ft. lumber at \$15 per M.
45. Of 1620 ft. lumber at \$16 per M.

(kk) Find the sum of the three results.

46. Of 99 lb. butter at 23¢ a pound.
47. Of 99 lb. butter at 28¢ a pound.
48. Of 98 lb. butter at 24¢ a pound.

(ll) Find the sum of the three results.

49. Paid \$15.50 for ribbon at $16\frac{2}{3}$¢ a yard. How many yards did I buy?

50. Paid \$24.75 for ribbon at $12\frac{1}{2}$¢ a yard. How many yards did I buy?

(mm) Find the sum of the two results.

EXAMINATION PROBLEMS.

Miscellaneous Problems.

NOTE.—The following problems are selected mainly from sets of **examination questions** supplied to the author for this purpose by one hundred school principals and superintendents.

1. Each edge of a cube is diminished by $\frac{1}{10}$ of its length:

(a) By what fraction of itself is the volume diminished?

(b) By what fraction of itself is the surface diminished?

2. How many cubical blocks, each edge of which is $\frac{1}{2}$ ft., are equivalent to a block 8 ft. long, 4 ft. wide, and 2 ft. thick?

3. A ladder 78 ft. long stands perpendicularly against a building. How far must it be pulled out at the foot that the top may be lowered 6 ft.?

4. A merchant sold $\frac{2}{3}$ of a quantity of cloth at a gain of 20% and the remainder at cost:

(a) His gain was what per cent of the cost?

(b) If he gained $7.29 what was the cost of the goods?

5. What must I pay for 4% stock to get 5% on the investment?

6. The cubical content of one cube is eight times that of another:

(a) How does an edge of the first compare with an edge of the second?

(b) How does the surface of the first compare with the surface of the second?

7. A creditor receives $1.50 for every $4.00 that is due him and thereby loses $301.05:

(a) What was the sum due him? (b) What per cent of the debt did he lose?

8. At $20 per M., board measure, what is the cost of the following: A stick of timber 30 feet long and 14 inches square, and a plank 18 feet long, 8 inches wide, and $2\frac{1}{2}$ inches thick?

9. A and B hire a pasture for $85; A puts in 8 cows and B puts in 12 cows. How much should each pay?

10. Simplify the following: $\frac{1}{3}$ of $\frac{7}{8}$ of $1\frac{1}{7}$.

11. Seven times John's property, plus $32200, equals 21 times his property. How much is he worth?

12. Two men engage in business with a joint capital of $5000. The first year's gain was $1760, of which one received $1056. How much capital did each furnish?

13. Thirty-five per cent of the men in a regiment being sick, only 637 men were able to enter battle. How many men were there in the regiment?

14. A lawyer collected 80% of a debt of $2360 and charged 5% commission on the sum collected. How much did the creditor receive?

15. Write a negotiable note for $500, making yourself the payee and James J. Rogers the maker. Interest at the legal rate.

16. A speculator bought stock at 25% below par and sold it at 20% above par. He gained $1035. How much did he invest?

17. What is the rate per cent per annum, if $712 gains $142.40 in 3 yr. 4 mo.?

18. A person asked for a lot of land, 40% more than it cost him, but finally reduced his price 15% of his asking price and sold it making $9.50:

(a) What per cent did he make? (b) How much did the land cost him? (c) How much did he receive for it?

19. Purchased stock at a premium of 8 per cent. What rate of interest do I receive on the investment if it pays an annual dividend of 6%?

20. Find the volume of a cube the area of whose surface is 100.86 square inches.

21. How many apples must a boy buy and sell to make a profit of $9.30, if he buys at the rate of 5 for 3¢ and sells at the rate of 4 for 3¢?

22. Find the cost of 1875 lb. hay at $6.50 per ton.

23. What is the interest on $1200 from Sept. 21, 1898, to May 5, 1899, at 7% per annum?

24. The area of a square field is 10 acres. What is the distance diagonally across the field?

25. A "drummer" earns $2500 a year. One thousand dollars of this sum is a guaranteed salary. The remainder is his commission of 5% on his sales. What is the amount of his annual sales?

26. The area of a triangle is 325 square inches. Its base is 25 inches. What is its altitude?

27. Gave $6\frac{2}{3}$ lb. butter worth 36¢ a pound for $3\frac{1}{3}$ gal. oil. What was the cost of the oil per gallon?

28. A can build a certain wall in 10 days; B can build it in 12 days, and C, in 15 days. In how many days can they build the wall working together?

29. A horse and a carriage together cost $550. The horse cost $\frac{3}{8}$ as much as the carriage. Find the cost of each?

30. A man willed $\frac{1}{3}$ of his property to his wife, $\frac{1}{3}$ of the remainder to his daughter, and the rest to his son. The difference between the wife's portion and the son's portion was 12480.33\frac{1}{3}$. How much was the man worth?

31. (a) How many loads, each containing a cubic yard, will be required to fill a street 150 feet long, 50 feet wide, and $2\frac{1}{2}$ feet deep? (b) How much will it cost at 18¢ per cubic yard?

32. A right triangle has two equal sides. Its hypothenuse is 100 rods long. (a) Find one of its two equal sides. (b) Find its area.

33. What is the ratio of the area of a circle to the area of its circumscribed square?

34. What is the ratio of the square of the radius to the square of the diameter of the same circle?

35. A man's tax is $37.50. The rate of tax is 1½%. Property is assessed at 30% of its value. What is the man's property worth?

36. A field containing 160 acres is 40 rods wide. At 45¢ a rod, how much less would it cost to fence a square field containing the same number of acres?

37. My agent in Baltimore having sold a consignment of grain, after taking out his commission at 3%, and paying a freight bill of $1125.00, sent me a draft for the amount due me—$19536.00. For how much was the grain sold?

38. How many pickets 4 inches wide, placed 3 inches apart, are required to fence a garden 21 rods long and 14 rods wide?

39. At $6.30 a cord what is the value of wood that can be piled under a shed 50 ft. long, 25 ft. wide, and 12 ft. high?

40. Find the curved surface of a cylinder 6 ft. in diameter and 12 ft. long.

41. (a) What is the bank discount and (b) what are the proceeds on a note for $125 payable in 90 days, the rate of discount being 8%?

42. If sugar that cost 5¢ a pound is sold at 18 lb. for a dollar, what is the gain per cent?

SET I.

COOK COUNTY, ILLINOIS.

Eighth Grade Examination for County Superintendent's Diploma.

June, 1897. O. T. BRIGHT, Supt.

Time, 9:30 to 12.

1. What is the ratio of (a) .2% to 2%? (b) 5 ÷ .5 = ? (c) .05 ÷ 5 = ? (d) .5 ÷ .05 = ? (e) .005 ÷ .5 = ? (f) .05 ÷ .005 = ?

2. A girl spelled correctly 95% of 60 words. How many words did she miss?

3. When a vessel sails 168 miles a day, she completes her voyage in 14 days. In what time would she complete it if she sailed 196 miles a day?

4. A three inch cube was painted on all sides. It was then cut into inch cubes. (a) How many of the inch cubes were painted on three sides? (b) How many on two sides? (c) How many on one side? (d) How many were not painted at all?

5. Fill the blanks in the following:

A boy had a fish pole 15 feet long. A piece equal to 20% of its length was broken off while catching fish.

(a) The part remaining was ——— % of the whole pole.

(b) The part broken off was ———% of the part remaining.

(c) The part remaining was ——— % of the part broken off.

6. The N.E. ¼ of the N.W. ¼ of a certain section of land was fenced off into four equal fields. (a) What is the shortest length of fence necessary? (b) How much land in each field?

7. A straight pole 72 feet high is broken 20 feet from the ground, but is not detached. How far from the foot will the top reach?

SET II.

NEW HAVEN (CONN.) PUBLIC SCHOOLS.

Extrance Examination to the High Schools.

Fall, 1897. C. N. KENDALL, Supt.

(Answer 10 *entire* questions.)

1. (a) 13 oz. is what per cent. of 5 lb. avoirdupois?

(b) A man added 18 cows to his herd, thereby increasing the number 25 per cent. How many cows has he now?

2. If lead pencils that cost 3 cents each, are sold for 5 cents each, what is the per cent. of profit?

3. Find the interest on a note for $330 at 6 per cent., given August 3, last year, and due to-day.

4. Write a negotiable, interest-bearing, promissory note.

5. (a) Divide $\frac{3}{4}$ of $\frac{5}{6}$ of $7\frac{1}{5}$ by $3\frac{3}{5}$.

(b) Subtract $8\frac{3}{16}$ from the sum of $5\frac{1}{3}$, $2\frac{1}{6}$, $4\frac{7}{12}$.

6. A rectangular field is $86\frac{1}{2}$ rods long and 46.875 rods wide. How much wheat will it produce at the rate of 20 bushels per acre?

7. A rectangular park, the sides of which are respectively 45 rods and 60 rods long, has a walk crossing it from corner to corner. How long is the walk?

8. If $\frac{3}{4}$ of 9 bushels of wheat cost $13\frac{1}{2}$, what will $\frac{5}{8}$ of a bushel cost?

9. If hay sells for $14 a ton at a loss of $12\frac{1}{2}$ per cent., what must it sell for to gain 15 per cent?

10. How many pounds of cotton at $7\frac{1}{4}$ cents a pound can a broker buy for $9225, and retain his commission of $2\frac{1}{2}$ per cent.?

11. Find the proceeds of a 3 months' note for $500 discounted at a bank at 6 per cent.

12. If a building 20 feet high casts a shadow of 6 feet, what length of shadow will a church spire 114 feet high cast?

SET III.

COOK COUNTY, ILLINOIS.

Applicants for Teachers' Second Grade Certificates.

June, 1898. O. T. Bright, Supt.

"Mental Arithmetic."

Time, 20 minutes.

Fill the blanks:

1. By a sale of goods, I lost $12\frac{1}{2}\%$. The cost was —— % of the selling price.

2. The circumference of a 3 in. circle is —— % of its radius.

3. A five inch square is —— % greater than a four inch square.

4. I buy apples four for three cents and sell them three for four cents. I gain —— per cent.

5. A and B are 78 miles apart and walk toward each other; A walks 3 miles an hour and B $3\frac{1}{2}$ miles an hour. When they meet B will have walked —— miles.

6. A stick of timber 15 inches square and 32 feet long contains —— board feet.

7. A horse trots $23\frac{3}{4}$ miles in $2\frac{1}{2}$ hours. His rate per hour is —— miles.

8. $154 was divided among A, B, C, and D, in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$. C got —— dollars.

9. A school contains 50 pupils;

Monday, 3 were absent in the forenoon.

Tuesday, 2 were absent all day.

Wednesday, 3 absent forenoon and 2 afternoon.

Thursday, 4 absent all day.

Friday, all present.

The per cent. of attendance for the week was ——.

10. 64 gal. of wine and 16 gal. of water were mixed. One pint of the mixture contained —— of a gal. of water.

SET IV.

COOK COUNTY, ILLINOIS.

Applicants for Teachers' Second Grade Certificate.
August, 1895. O. T. Bright, Supt.
Time, 60 minutes.

1. If a quarter section of land has fenced within it the largest possible circular lot, how many acres of the quarter section will remain outside of the circle?

2. Find the value of the following lumber at $15 per M.:

20 pieces 2x4, 18 ft. long.
20 pieces 4x4, 12 ft. long.
20 pieces 3x10, 16 ft. long.
45 16 ft. stock boards, 15 inches wide.

3. (a) What sum invested in 8% bonds at 33⅓% premium will yield an annual income of $1200?

(b) What if the bonds were 33⅓% discount?

4. Find the value of a piece of land 20 ft. x 40 rods at $1000 per acre.

5. What is the ratio of $3\frac{1}{8}$ to $\frac{5}{6}$? Answer in per cent.

SET V.

Examination Department of the University of the State of New York, January, 1898.
100 credits. Necessary to pass, 75.
Time, 9:15 a. m. to 12:15 p. m.

Answer the first five questions and five of the others, but *no more.* If more than five of the others are answered only the first five answers will be considered. Give all operations (except mental ones) necessary to find results. Reduce each result to its simplest form and mark it *Ans.* Each complete answer will receive 10 credits.

1. Define numerator, denominator, divisor, factor, proportion.

2. Find the weight in kilograms of a stone 1 meter

square and .4 of a meter thick, assuming that the stone is $2\frac{1}{2}$ times as heavy as water.

3. Simplify $\frac{3\times\frac{2}{7}\times 4.2}{\frac{5}{18}\times\frac{2}{5}\frac{0}{7}}$

4. Find the interest on \$375 at $4\frac{1}{2}\%$ from July 1, last year, to the present time.

5. Multiply 65.15 by 3.14159 and divide the result by 57.296, finding a result correct to three decimal places.

6. Find the cost at \$50 an acre of a rectangular field 1650 feet long and 825 feet wide.

7. Find the time required to fill a cistern 8 feet square and 5 feet deep by a pipe which admits water at the rate of 1 quart a second.

8. Make a receipted bill of the following: J. L. Robbins & Co. sold this day to Samuel Jones 8 yards of cloth at $37\frac{1}{2}$ cents, 24 yards of calico at $8\frac{1}{3}$ cents, 1 dozen handkerchiefs at $12\frac{1}{2}$ cents each, and 3 dozen towels at \$2.50 a dozen.

9. Find the cost of four sticks of timber, each 8 inches by 10 inches and 30 feet long, at \$15 a 1000 feet board measure.

10. Find the least common multiple of 153, 204, and 510.

11. If 4% bonds to the amount of \$8000 face value are bought at $92\frac{1}{2}\%$, find the cost of the bonds, and the rate of income on the investment.

12. If 3 men can do a piece of work in 8 days of 10 hours each, how many men will be required to do the same work in 6 days of 8 hours each? (Solve by proportion.)

13. By selling a horse for \$144, a profit of 60 per cent is made; find the cost of the horse.

14. The diameter of a bicycle wheel is 28 inches; find the number of revolutions it makes in going 1 mile.

15. Find the square root of 7, correct to three decimal places.

SET VI.

Examination for Admission to State High Schools.

Minnesota, 1898.

Time, 2 hours.

Answer any six—no more If more are attempted and the student does not designate which six he wishes to be graded upon, the first six answers will be taken.

1. a (2) What is the ratio of 2 to .90?

   b (2) What is the ratio of $\frac{3}{4}$ to $\frac{3}{10}$?

   c (2) What is the ratio of $\frac{3}{10}$ to $\frac{3}{4}$?

   d (2) What is the ratio of 90 % to .09?

   e (2) What is the ratio of 70 % to 50 %?

2. a (5) What is the cubical content of a cellar 15 ft. wide, 20 ft. long, and 10 ft. deep? (In the solution express all operations in the form of equations.)

   b (1) What unit (or units) of measure did you use in the example?

   c (4) Describe the unit of measure used in measuring boards.

3. (10) A merchant sells an overcoat for $22; a suit of clothes for $23; a hat for $5. On the overcoat he makes 10 % of the cost; on the suit 15 %, and on the hat 25 %. What per cent of the cost of the goods does he make on the entire sale?

4. a (5) A man bought a watch and a chain for $70. One-half of the cost of the watch equals $\frac{2}{3}$ of the cost of the chain. What was the cost of each?

   b (5) Analyze.

5. (10) The rates at which A, B, and C work are to each other as 2, 3, and 4. What integers will indicate the time it will take each to do a certain piece of work?

6. (10) How long a rope must a horse have in order that he may graze over an acre of land, if he be tied to a stake in the center of a field?

7. (10) B buys bank stock at 78 and sells it at 84. C buys railroad stock at 70 and sells at 75. Each buys the same number of shares, and B makes $1000 more than C. How much money did B invest?

8. (10) Make (5) and solve (5) a problem in the solution of which it will be necessary to extract the square root.

9. a (5) What is the interest on $700 for 1 yr. 5 mo. and 10 da. at 7 % per annum.

b (5) Analyze.

10. (10) A servant is engaged for a year for $280 and a suit of clothes. He leaves at the end of six months and receives $130 and the suit. What is the value of the suit? (An algebraic solution is allowed for this problem.)

11. a (5) How long will it take $1560 at 5 % simple interest to gain $426.83⅓?

b (5) Analyze.

SET VII.

Examination for State Certificates.

Illinois, 1898.

Time, two hours.

1. (a) Every fraction is a ratio. Explain.
   (b) Every integral number is a ratio. Explain.

2. In the report of the Committee of Ten it is recommended that "the course in arithmetic be at the same time abridged and enriched."
   (a) Tell what abridgment you regard as important.
   (b) Tell what enrichment you consider essential.

3. (a) Tell what sense-magnitudes you prefer to use in presenting to third grade pupils the subject of fractions. (b) At what stage of the work do you think sense-magnitudes should give place to imaginative magnitudes?

4. (a) When and to what extent should pupils in the grades be required to memorize definitions of mathematical terms? (b) When and to what extent should pupils be required to memorize directions for performing operations?

5. Mention all the standard linear units with which you are familiar, and give the ratio of each (either exact or approximate) to some other linear unit.

6. What is the weight of 1000 feet of white pine boards (1 inch in thickness) if the specific gravity of the boards is .6?

7. The foundation of my house is 32 feet square on the outside. The house is 20 feet high to the plates and the roof has the usual eave-projections. Give approximately the number of barrels (31½ gal.) of water that will fall upon this roof in one year, if the rain-fall is 34½ inches.

8. Give approximately the following ratios:

(a) Of the circumference to the diameter of a circle.
(b) Of the diagonal to the side of a square.
(c) Of the area of a circle to the area of its circumscribed square.
(d) Of the area of a circle to the area of its inscribed square.

9. What single discount is equal to a discount of 45 per cent. and "5 10's", *i.e.*, to "45 and 10 and 10 and 10 and 10 and 10 off", from the list price?

10. If money is worth 6 per cent. annual interest now and prospectively, what is the actual cash value of a note of $1000 running two years and drawing 5 per cent., interest payable annually?

VIII. The Bank Test.*

To THE TEACHER.—Below are figures representing 51 sums of money. Procure 51 blank checks and cause them to be filled, using the sums here given. Draw one check from the 51 checks and give the remaining 50 to a pupil to transcribe the sums and find their amount. When the pupil obtains a result the teacher can quickly determine whether it is correct by comparing it with the sum of the 51 checks, *less the sum named on the check drawn out.* Before the checks are given to the second pupil, the check removed should be replaced and another withdrawn. Thus, although each pupil should obtain a result differing from that obtained by the pupil preceding him, its accuracy can be quickly tested by the teacher.

To THE PUPIL.—Can you, *on first trial,* transcribe the sums named on 50 checks and find the amount accurately in 30 minutes?

$324.56 $565.60 $123.20 $75.00
$234.50 $525.40 $312.95 $190.35
$46.45 $112.00 $86.50 $250.00
$325.00 $86.74 $91.23 $50.00
$302.26 $59.29 $12.65 $8.25
$7.75 $875.00 $1.50 $431.05
$201.45 $34.36 $85.40 $90.00
$130.25 $212.24 $230.94 $642.45
$71.20 $708.30 $60.00 $75.00
$1250.25 $6.50 $500.00 $2324.45
$9.10 $101.50 $36.09 $275.00
$150.00 $2.50 $1008.60 $140.65
$256.74 $987.84 $50.00

* In a leading bank in Chicago, it is customary to test applicants for positions as accountants by placing before them 150 checks, requiring each applicant to copy the sums named on the checks and find their amount. The author of this book is informed that the average inexperienced applicant does this in about 30 minutes, with some errors, however, both in transcribing and in footing. An expert accountant can do this amount of work *accurately* in 6 minutes.

IX. Curious Comparisons.

1. If a pig whose girth is 2 feet, weighs 50 lb., what is the weight of a similarly proportioned pig whose girth is 4 feet?

2. If a disk of dough 15 inches in diameter is sufficient for 20 doughnuts, how many such doughnuts can be made from a disk 30 inches in diameter?

3. The bore of a 10-inch gun is how many times as large as the bore of a 2-inch gun?

4. The ball of a 10-inch gun is how many times as large as the ball of a 2-inch gun?

5. A square, a pentagon, a hexagon, an octagon, and a circle have equal perimeters. (a) Which has the greatest area? (b) Which has the least area?

6. The capacity of a cistern 6 feet in diameter and 6 feet deep is about 40 barrels. What is the capacity of a cistern 12 feet in diameter and 12 feet deep?

7. A $2\frac{1}{2}$-inch pipe is how many times as large as a 1-inch pipe?

8. If a man 6 feet tall weighs 190 lb., how much would a similarly proportioned giant 12 feet tall, weigh?

9. In a certain orchard the trees are 15 feet apart each way and there are 800 trees. How many trees in an orchard of equal size, the trees being 30 feet apart each way?

10. A ball of yarn 3 inches in diameter is sufficient for one mitten. How many mittens can be made from a ball 6 inches in diameter?

11. A grindstone was originally 30 inches in diameter. It has been worn until it is but 15 inches in diameter. What part of the stone has been worn away?

12. A square and an oblong have equal areas. Which has the greater perimeter?

X. Practical Approximations.

Without the aid of a pencil (except for writing results) give approximate answers, sufficiently accurate for many practical purposes, to the following problems:

1. What is the area of a circle 20 inches in diameter?
2. What is the volume of a 12-inch globe?
3. What is the circumference of a circle that is 40 rods in diameter?
4. What is the capacity, in gallons, of a tank 6 ft. by 5 ft. by 2 ft.?
5. Is the capacity of a cylindrical pail 6 inches in diameter and 5 inches deep, more or less than $\frac{1}{2}$ a gallon?
6. A piece of land 41 rods by 24 rods is how many acres?
7. What is the capacity in barrels ($31\frac{1}{2}$ gal.) of a tank 4 ft. by 4 ft. by 2 ft.?
8. What is the weight of one cord of green hickory wood?
9. What is the weight of a stick of seasoned white pine timber 12 in. by 12 in. and 30 feet long?
10. What is the weight of the water that will fill a cylindrical tank that is 8 feet in diameter and 8 feet deep?
11. A circular $\frac{1}{2}$-mile race track encloses how many acres?
12. If the six-foot drive wheel of a locomotive makes 840 revolutions in moving a certain distance, how many revolutions will a seven-foot wheel make in moving the same distance?
13. Twenty-four kilometers is how many miles?
14. Forty-two kilograms is how many pounds?
15. A 2-inch rainfall makes how many barrels of water to the acre?
16. A certain roof is 40 feet long and, measured horizontally, 24 feet in width. A 2-inch rain-fall should give how many inches in depth in a cistern that receives the water from this roof, the cistern being 6 feet long and 4 feet wide?

XI. Puzzling Problems.

1. If a person traveling as expeditiously as possible from Boston to San Francisco, should mail a letter to his friend in Boston every day at noon, how often would the letters be received in Boston?

2. If a man and a boy, the boy doing exactly one-half as much work as the man, can hoe one and one-half acres of corn in one and one-half days, how many acres can 6 men hoe in 6 days?

3. John and James sold apples together. The first day they sold 60 apples at the rate of 5 apples for 2 cents, and received 24 cents. The second day they divided the apples. John took 30 of the larger apples and sold them at the rate of 2 for 1 cent. James took the remaining 30 apples and sold them at the rate of 3 for one cent. They received 25 cents. Why did they receive one cent more the second day than the first?

4. A pile of four-foot wood stands upon a hill-side. The pile is 8 feet long (measured on the ground), and 4 feet high (measured vertically). Does the pile contain one cord?

5. A man had shingles enough to cover his house if he laid them 4 inches to the weather. He laid them $4\frac{1}{2}$ inches to the weather. What part of the shingles provided remained? Explain.

6. If on a line of railroad connecting Chicago and San Francisco one passenger train leaves Chicago daily at 6 o'clock A.M., and makes the journey to San Francisco in exactly five days, and one train leaves San Francisco daily at 6 o'clock P.M., and makes the journey to Chicago in exactly five days, (a) a person taking the train at Chicago will meet how many passenger trains while going to San Francisco? (b) How many trains of passenger cars required to equip the road?

BUSINESS PROBLEMS.

The Extension Clerk.

NOTE.—In large wholesale houses, clerks are sometimes employed in *extending bills.* These clerks receive the bills upon which the amount, description, and price of the goods sold, are given, and *extend* each line; that is, they carry into the proper place the value of each purchase. They also add the columns of figures representing these values.

CHICAGO, June 12, 1901.

Illinois School for the Blind,
Jacksonville, Ill.

Bought of DURAND & KASPER CO.

				A		B	
1	*box Oatmeal Wafers*	26½	.07½	* –	–		
1	*Cheddar Cheese*	45	.12½	–	–		
1	*bag Rio Coffee*	150	.16½	–	–		
1	*bbl. Granulated Sal Soda*	325	.70	† –	–		
5	*cases Elgin Corn*	5/2	1.10	‡ –	–		
8	" *Columbian Corn*	8/2	.80	–	–		
6	" *Wayne Co. Peas gal*	5/1	4.60	–	–		
2	" *2 lb. Salmon*	2/2	2.85	–	–		
3	" *Tomatoes gal*	3/1	2.50	–	–		
1	" *Col. Riv. Salmon*	1/4	1.40	–	–		
1	*box Oyster Crackers*	22	.06½	–	–		
½	*chest Japan Tea*	40	.35	–	–		
5	*bags Navy Beans*	764	2.00	§ –	–		
5	*boxes Bulk Starch*	250	.03½	–	–		
1	*box Bulk Pepper*	12	.14	–	–		
1	*bbl. White Eagle Drips*	54½	.16	–	–		
1	" *Yellow Corn Meal*	1	2.30	–	–		
1	*box Tapioca*	25	.05⅛	–	–		
5	*bbl. Gran. Sugar* 346 344 340 362 334	1726	5.51	‖ –	–	–	–

1. Copy and extend the above bill. The dashes indicate where the extension figures should appear. Put the amount of the bill, in column B opposite the last line in column A.

* 26½ lb. @ 7½¢ a pound. † 325 lb. @ 70¢ per cwt. ‡ 5 cases; 2 doz. in a case; $1.10 a dozen. § 764 lb. @ $2.00 a bushel. ‖ 1726 lb @ $5.51 per cwt.

CHICAGO, June 13, 1901.

Mr. Ralph T. Dodge,
Earlville, Illinois.

Terms: 10 days less 6%
30 days less 5%
60 days less 3%
After 60 days, Net.

Bought of A. C. McCLURG & Co.

Salesman, *Gould.*

			A	B
1	*Ebers*	*In the Desert*	1 50	
2	*Ruskin*	*Stones of Venice* 4.50	* – –	
2	"	*Seven Lamps of Architecture* 1.50	– –	
1	*Coolidge*	*What Katy Did* 1.25 15%		† – –
1	*Herrick*	*Web of Life*	1 50	
2	*Warner*	*Golden House* 2.00	– –	
2	*Potter*	*Uncanonized* 1.50 20%		– –
2	*Thompson*	*Political Economy* 1.00 6%		– –
3	*Steel*	*Hosts of the Lord* 1.50	– –	
2	*Besant*	*Fourth Generation* 1.50	– –	
1	*Garland*	*Eagle's Heart*	1 50	
2	*Seawell*	*House of Egremont* 1.50	– –	
2	*Harris*	*On the Wing of Occasions* 1.50	– –	
1	*Taylor*	*Cobbler of Nimes* 1.25 20%		– –
2	*Putnam*	*Book of Common Worship* 1.25 10%		– –
4	*Ade*	*Fables in Slang* 1.00	– –	
2	*Ward*	*Eleanor* 1.50	– –	
2	*Mulock*	*A Hero* .90	– –	
1	*Sheridan*	*The Rivals*	75	
			– –	
		Less 25%	‡ – –	‖ – –
				– –

1. Copy, extend, and find the amount of the above bill.
2. How much must Mr. Dodge remit if paid in 10 days?
3. In 30 days? 4. In 60 days? 5. In 75 days?

NOTE.—In this bill the *list price* of all books upon which the discount is exactly 25%, should appear in column A. The *net price* of all other books is carried directly into column B. The sum of column A less 25% of itself is carried into column B. The sum of column B is the *amount* of the bill; but a special discount, as shown at the top of the bill, is allowed for prompt payment. The dashes indicate where the extension figures should appear.

* Two books @ $4.50 each.
† $1.25 less 15% of itself.
‡ 25% of the footing of column A.
‖ The footing of column A less 25% of itself

Dr. J. C. Gordon,
Jacksonville, Ill.

CHICAGO, June 14, 1901.

Bought of JAMES B. CLOW & SONS.

Salesman, *Hamilton.*

			Price	A		B	
24	½″	C. I. Ells	.06	* –	–		
36	¾″	" " "	.08	–	–		
36	1″	" " "	.10⅓	–	–		
12	1¼″	" " "	.20	–	–		
12	2″	" " "	.28	–	–		
6	3″	" " "	.75	–	–		
24	¾″	" " Tees	.12	–	–		
36	1″	" " "	.15	–	–		
		30, 10 & 5% off		*a* –	–	*b* –	–
6	1½″	*Flange Unions*	.78	–	–		
12	2″	" "	1.00	–	–		
6	2½″	" "	1.25	–	–		
3	4″	" "	2.10	–	–		
		35% off		*a* –	–	*b* –	–
† 507′	0″	¾″ *Black Pipe*	.11⅓	–	–		
519′	0″	1″ " "	.16⅔	–	–		
209′	4″	1¼″ " "	.27	–	–		
101′	1″	3″ " "	.75½	–	–		
		38.2% off		*a* –	–	*b* –	
6	¾″	*Globe Valves*	2.20	–	–		
1	3″	" "	16.00	–	–		
		50 & 10% off		*a* –	–	*b* –	–
		Boxing & Drayage					45
						–	–

1. Copy, extend, and find the amount of the above bill.

NOTE.—In this bill *list prices* are given. In column A, values according to list prices are to appear; in column B, net values. The dashes indicate where the extension figures should appear.

* 24 ½-inch cast iron ells at 6¢ each. † 507 feet, 0 inches.

a & *b*. The footing should appear at *a*; the footing less the discount, at *b*.

CHICAGO, June 15, 1901.

Mr. J. H. Collins,
Springfield, Illinois.

Terms: 10 days less 2%
30 days less 1%
60 days no discount.

Bought of MARSHALL FIELD & CO.

Salesman, *Brett.*

			Am't	Price	A		B	
*	1	*Muslin ..... Old Glory*	52	.06	*a* –	–		
	1	" ..... *Dwight*	52	.07½	*b* –	–		
	1	" ..... *Aurora*	52	.03¾	*c* –	–	*d* –	–
	1	*Calico ..... Katrina*	44	.05	–	–		
	1	" ..... *Simpson*	42	.04¾	–	–	–	–
	3	*Gingham 32 30 26*	88	.08	–	–	–	–
†	4	*Knitting Cotton*	4	.65	*e* –	–		
		30 & 10% off			*f* –	–	*g* –	–
	7	*Coats Thread*	7	.55	*e* –	–		
		7% off			*f* –	–	*g* –	–
‡	2	*Towel ..... No. 726*	2	1.37½	–	–		
‖	6	*Crash 6/40*	240	.12¼	–	–	–	–
	2	*Brush ..... No. 3642*	2	1.15	–	–		
	2	*Comb ..... No. 7260*	2	1.05	–	–	–	–
							–	–

1. Copy, extend, and find the amount of the above bill.
2. How much must Mr. Collins remit if paid in 10 days or less?
3. How much if paid in 30 days?
4. How much if paid in 60 days?

* 1 piece of muslin containing 52 yd. @ 6¢ a yard.
† 4 dozen balls @ 65¢ a dozen.
‡ 2 " towels, No. 726, @ $1.37½ per dozen.
‖ 6 pieces crash, 40 yd. in each piece.
a, *b*, *c*, *d*, etc. The sum of *a*, *b*, and *c* should appear at *d*; *f*, is the discount; *g* is *e* less the discount.

Tax Extension.

Tax extension clerks compute what each property holder must pay for the several purposes for which taxes are exacted. To aid in doing this work extensive tables are usually provided by which the tax on any assessed value at any give rate may be found without arithmetical process,—by mere reference to the tables. Such tables cannot be presented within the limits of an ordinary arithmetic.

In the tax extension required on page 292, the following substitute for these tables may be employed to advantage. This table shows the tax on any number of hundred dollars from 1 to 9. It is apparent that the tax on any number of thousands of dollars would be ten times as much as the amount shown in the table; on any number of tens of dollars, one tenth as much, etc. "R. & B." stands for *Road and Bridge;* "Lib.," for *Library.*

TAXES IN SCHOOL DISTRICT NO. 4, AURORA, ILL.

Assessed Value.	State.	County.	Bond.	Town.	R. & B.	Lib.	City.	School.	Total.
$100	.50	.75	.16	.08	.35	.15	2.21	2.31	6.51
$200	1.00	1.50	.32	.16	.70	.30	4.42	4.62	13.02
$300	1.50	2.25	.48	.24	1.05	.45	6.63	6.93	19.53
$400	2.00	3.00	.64	.32	1.40	.60	8.84	9.24	26.04
$500	2.50	3.75	.80	.40	1.75	.75	11.05	11.55	32.55
$600	3.00	4.50	.96	.48	2.10	.90	13.26	13.86	39.06
$700	3.50	5.25	1.12	.56	2.45	1.05	15.47	16.17	45.57
$800	4.00	6.00	1.28	.64	2.80	1.20	17.68	18.48	52.08
$900	4.50	6.75	1.44	.72	3.15	1.35	19.89	20.79	58.59

EXAMPLES.

PROBLEM 1.

On $600, $2.10
" 60, .210
" 6, .0210

$2.331

or

.35% of $666.
Tax, $2.34*

1. What is the tax at 35¢ per $100 on property the assessed value of which is $666?

2. What is the tax at $2.21 per $100 on property the assessed value of which is $958?

PROBLEM 2.

On $900, $19.89
" 50, 1.105
" 8, .1768

$21.1718

or

2.21% of $958.
Tax, $21.18*

*In many states it is provided by law that "*In the extension of taxes, the fraction of a cent shall be extended as 1 cent.*"

TAXES IN SCHOOL DISTRICT NO. 4, AURORA, ILLINOIS.

	Value.		State.	County.	Bond.	Town.	R. & B.	Lib'y.	City.	School.	Total.
A	736	00									
B	845	00									
C	2621	00									
D	524	00									
E	1389	00									
F	621	00									
G	235	00									
H	5175	00									
I	811	00									
J	375	00									

1. Make an enlarged copy of the above upon which extend the taxes of A, B, C, etc.

NOTE.—It will be well for the student to find each tax item (1) by means of the table given on the preceding page and (2) by finding the product of the assessed value and the rate. The two results should be identical.

Find the total of each person's tax by adding the eight tax items. Prove the work by finding the tax of each person, at $6.51 per hundred dollars of assessed valuation. The amount thus found will be a few cents less than the amount found by adding. See foot-note on page 291.

In the Bank.

Interest on Daily Balances.

Sometimes a person who has a large but constantly varying sum of money which he wishes to keep on deposit in some reliable bank, can arrange with the bank officials to receive at the end of each month 2% or more on his daily balances. The accountant at the bank ordinarily employs carefully prepared tables for computing the interest due a depositor. Regarding 360 days as a year, accounts like the following can easily be computed without the aid of interest tables.

DAVID EUGENE SMITH.

1901	Debit.	Credit.	Balance.	Days.	Interest.
April 1		5000	5000	4*	20000
" 5		575 25	5575 25	5†	27875
" 10	145 20		5430 05	1	5430
" 11	200	600	5830 05	9	52470
" 20	432 50	100	5497 55	10	54970
" 30		650	6147 55		160745‡
" 30	Interest Credit 8 93		6156 48	2	
May 2		800			
" 6	150				
" 10	300	500			
" 14	246 25				
" 19	100	750			
" 28		564 75			
" 30	261 42	100			
" 31	Interest Credit				

1. Make an enlarged copy of the above account and extend the balances and interest for the month of May. Compute the interest at the rate of 2% per annum.

* Mr. Smith is entitled to interest on $5000 from Apr. 1 to Apr. 5, 4 days. (5000×4 =20000). This equals the interest on one dollar for 20000 days. Hence "20000" is carried into the interest column.

† In computing interest on daily balances, it is customary to omit any fraction of a dollar that may appear in the principal. In some banks interest is computed on full hundreds of dollars only; thus, if the balance is $5742 interest is allowed on $5700.

‡ The interest on $1.00 for 160745 days at 6% is (160745 ÷ 6) 26790 mills; at 2% it is one third this number (26790÷3) or 8930 mills. This equals $8.93.

6)160745‡
3)26790 mills.
8930 mills.

Interest on Daily Balances.

Rule paper as suggested on the preceding page, arrange in proper form the accounts given below, and find the balance of each account on the last day of the month, allowing interest on daily balances at the rate of 2% per annum.*

1.

FLORIAN CAJORI.

June 1, deposited $8000. June 6, deposited $400.
June 10, deposited $500 and withdrew $175.50.
June 18, withdrew $248.50. June 20, withdrew $675.20.
June 25, deposited $100 and withdrew $241.20.
June 30, deposited $200.

2.

ARTHUR LEFEVRE.

May 6, deposited $4270. May 7, withdrew $342.50.
May 12, deposited $200 and withdrew $525.37.
May 31, deposited $500 and withdrew $1246.20.

3.

W. K. CLIFFORD.

July 4, deposited $25240.16. July 5, withdrew $1381.20.
July 6, withdrew $2375.50. July 7, withdrew $2150.45.
July 8, withdrew $537.25. July 9, withdrew $2150.45.
July 15, withdrew $146.31. July 25, withdrew $322.15.

4.

MCLELLAN AND DEWEY.

Aug. 10, deposited $10000. Aug. 15, deposited $5000.
Aug. 20, deposited $3000. Aug. 24, deposited $6000.
Aug. 30, deposited $1000 and withdrew $3246.54.

* These problems are mainly for a test of the ability of the student to work in figures with care and to secure accurate results. Let no pupil be satisfied with his self-training in this respect unless he can do all the work suggested on this page, and present it to his teacher in good form and absolutely free from errors in figures. Unless he can do this, he would, with his present attainments, be of little value as an accountant. The business world demands *accuracy* in accounts.

Compound Amount.

It is sometimes necessary in business to find the compound amount of money at interest. This is done by adding the interest to the principal at the end of a specified period and using the sum thus found as a new principal upon which the interest is computed and the amount obtained as before. This process is repeated as many times as there are interest bearing periods. To find the compound interest, subtract the original principal from the compound amount.

1. What is the compound amount of $200 for 2 years, interest at 6% per annum to be compounded annually?

2. What is the compound interest of $200 for 2 years, at 6% per annum to be compounded annually?

3. How much more is the compound interest of $200 for 2 years at 6%, than the simple interest of the same sum for the same time and rate? Why?

4. Without solving the two interest problems in full, tell how much more the compound amount of $500 for 2 years at 6%, is, than the simple amount of the same sum for the same time and rate.

5. Find the compound amount of $350 for 2 yr. 6 mo. at 6% per annum, interest to be compounded annually.*

6. Find the compound amount of $350 for 2 yr. 6 mo. at 6% per annum, interest to be compounded semi-annually.† Compare the answer to this problem with the answer to Problem 5.

7. Find the compound amount of 425.40 for 2 yr. 4 mo. 12 da. at 6%, interest to be compounded annually.

8. If Mark Mahan puts $100 at interest at the rate of 6% per annum, on the first day of each January for 8 successive years, and if at the end of each year he collects and loans the interest due him, at the same rate, all this will amount to how much, 8 years from the time of his first investment?

* Interest must be computed for three periods—the first two, one year each, and the third, six months.

† This calls for the compound amount for five periods at 3%.

Table

Showing the compound amount of $1.00 at 2, 3, 4, 5, and 6 per cent, for any number of periods, from 1 to 20.

	2 per cent.	3 per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.020,000	1.030,000	1.040,000	1.050,000	1.060,000
2	1.040,400	1.060,900	1.081,600	1.102,500	1.123,600
3	1.061,208	1.092,727	1.124,864	1.157,625	1.191,016
4	1.082,432	1.125,509	1.169,859	1.215,506	1.262,477
5	1.104,081	1.159,274	1.216,653	1.276,282	1.338,226
6	1.126,162	1.194,052	1.265 319	1.340,096	1.418,519
7	1.148,686	1.229,874	1.315,932	1.407,100	1.503,630
8	1.171,659	1.266,770	1.368,569	1.477,455	1.593,848
9	1.195,092	1.304,773	1.423,312	1.551,328	1.689,479
10	1.218,994	1.343,916	1.480,244	1.628,895	1.790,848
11	1.243,374	1.384,234	1.539,454	1.710,339	1.898,299
12	1.268,242	1.425,761	1.601,032	1.795,856	2.012,196
13	1.293,607	1.468,534	1.665,074	1.885,649	2.132,928
14	1.319,479	1.512,590	1.731,676	1.979,932	2.260,904
15	1.345,868	1.557,967	1.800,944	2.078,928	2 396,558
16	1.372,786	1.604,706	1.872,981	2.182,875	2 540,352
17	1.400,241	1.652,848	1.947,901	2.292,018	2.692,773
18	1.428,246	1.702,433	2.025,817	2.406,619	2.854,339
19	1.456,811	1.753,506	2.106,849	2.526,950	3.025,600
20	1.485,947	1.806,111	2.191,123	2.653,298	3.207,136

1. Solve again the problems on page 295, using in the work, items from the above table.*

NOTE.—In Savings Banks the interest due depositors is compounded usually semi-annually; that is, the interest if not withdrawn, is added to the principal every 6 mo.

2. Twelve hundred dollars deposited in a savings bank that pays interest at the rate of 4% per annum and compounds it every 6 mo., will amount to how much in 10 years?†

* To solve problem 8 on page 295, it will only be necessary to add the first eight numbers in the "6 per cent" column and multiply the sum by 100. Why?

† Think of this money as loaned for 20 "2 per cent" periods.

Present Value.

It is often important to know the exact present value of a *promise to pay* a sum of money after a specified number of years, months, or days. The rule for solving such problems is,—*Divide the amount to be paid by the amount of $1.00 for the given time and at the current rate.* Discuss the reasonableness and accuracy of this rule in its application to the special cases presented on this page.

1. What is the present value of a good note for $477, drawing no interest, and due one year hence, the current rate being 6%?

2. Prove your answer to Problem 1 to be correct by finding the amount of it for one year at 6 per cent.

3. What is the present worth of $500 due 2 years hence, the current rate for money being 6% interest payable annually?*

4. If a father should give to one son a note for $500 payable in 2 years without interest, and to another son, an amount of cash equal to your answer to Problem 3, and the second son should loan his money for two years at 6% with interest payable annually, and at the end of the first year should collect the interest due and loan it for one year at 6%, how would the amounts received by the two sons compare at the end of two years?

5. What is the present worth of a good note for $600, that matures 2 years hence and draws interest from the present time at 8%, the interest being payable annually, and the current rate for money being 6%, interest payable annually?†

6. What is the present value of a good note for $800, that matures 3 years hence, and draws interest from the present time at the rate of 4% per annum payable annually, the current rate for money being 6% interest payable annually?

*Since money is worth 6% interest payable annually, one dollar in two years would amount, not to $1.12, but to $1.1236. See table on preceding page.

†Observe that this problem calls for the present value of $48 due 1 year hence, and $648 due 2 years hence.

Investments.

1. The dividend on certain stocks being prospectively for all time, 8% per annum, how much shall one pay per share (par value $100) in order to realize 6% on the investment?

2. The current rate of interest continuing to be 6% for all time, would an 8% bond running 20 years be worth as much as the 8% stock mentioned in Problem 1?

3. The dividend on certain stocks being prospectively for all time, 5% per annum, how much shall one pay per share (par value $100) in order to realize 6% per annum on the investment?

4. The current rate of interest continuing to be 6% per annum for all time, would a 5% bond running 20 years be worth more or less than the 5% stock mentioned in Problem 3?

5. How much, including brokerage,* must one invest in 8% stocks at 38% premium in order to have an annual income of $1000?

6. How much, including brokerage, must one invest in 4% stocks at 28% discount in order to have an annual income of $1000?

7. A man invests (not including brokerage) $27,500 in bank stock at a premium of 10%; if the stock yields a semi-annual dividend of $3\frac{1}{2}$%, what is his annual income?

8. U.S. 4's, due in 10 years are bought at 8% premium. (a) What would be the cost (not including brokerage) of twelve $100 bonds? (b) What is the annual income on the twelve bonds?

9. To secure 5% on the investment, how much can one afford to pay for stock that yields an annual dividend of 6%?

10. To secure 5% on the investment, how much can one afford to pay for stock that yields an annual dividend of 4%?

*Stocks are often bought or sold through a broker. The commission for transacting the business is called the brokerage. The usual brokerage is $\frac{1}{8}$ of 1% reckoned on the par value, or $12\frac{1}{2}$¢ per $100 share.

Investments.

1. What is the average per cent of annual income on a 20-year 6% bond bought at a premium of 20% ? *

2. What is the average per cent of annual income on a 20-year 4% bond bought at a discount of 5% ?

3. What is the average per cent of annual income on a 25-year 5% bond bought at a premium of 2% ?

4. What is the average per cent of annual income on a 10-year 5% bond bought at a discount of 2%?

5. At what price must 6% 10-year bonds be bought to yield an average annual income of 5% ? †

6. The converse of the foregoing problem is as follows: What is the average per cent of annual income on a 10-year 6% bond bought at a premium of ——% ? ‡

7. At what price must 4% 10-year bonds be bought to yield an average annual income of 5% ?

8. The converse of the foregoing problem is as follows: What is the average per cent of annual income on a 10-year 4% bond bought at a discount of —— % ? §

9. If a man pays $1500 for a city lot from which he receives no income, but upon which he pays a tax of $20 a year (the first tax being paid exactly one year after purchasing the lot) for how much must he sell the lot immediately after paying his tenth annual tax, in order to realize 6% compound interest on his investment, regarding the amount paid for taxes as a part of the investment? ||

* A $100 6% bond bought for $120, in 20 years yields (20 times $6) $120 less the $20 of premium paid; or, $100. The average yield per year then is ($100 ÷ 20) $5. $5 is what per cent of $120 the amount originally invested?

† A $100 6% bond in 10 years amounts to $160. One dollar in 10 years at 5% amounts to $1.50. It will take as many dollars to amount to $160 as $1.50 is contained times in $160.

‡ Put the premium at which the bonds described in Problem 5, must be bought, in place of the dash.

§ Put the discount at which the bonds described in Problem 7, must be bought, in place of the dash.

‖ In solving this problem use the table on page 296.

Investments.

1. A house cost $5500. The average annual necessary expenditures are: Repairs, $120; taxes, $52; insurance, $12.50. The house rents for $35 a month. What per cent, net, does the owner realize on the investment?

2. A house cost $1000. The average annual necessary expenditures are: Repairs, $25; taxes, $12.75; insurance, $6.40. The house rents for $10 a month. What per cent, net, does the owner realize on the investment?

3. A house rents for $25 per month, and the owner pays $60 for taxes, insurance, and repairs. On what sum does the house pay a net profit of 5% per annum?

4. Three flats that cost $8500 rent for $35, $30, and $25 per month. If the flats on the average are vacant $\frac{1}{12}$ of the time and if the annual cost of repairs, insurance, taxes, water, fuel, and janitor services is $565, what is the net income on the investment?

5. A farm cost $10,000 including buildings that are valued at $3000. If the farm and buildings rent for $600 per annum, and the owner pays annually for repairs $62, taxes $55, and insurance $13; and if the annual depreciation on the value of the buildings is equal to 2% of their original value, what is the net per cent of income on the investment?

6. To make 5% on the investment, how much should be paid for a house that rents for $30 a month, if repairs, taxes, and insurance average $125 per annum?

7. Land that cost $70 per acre and rents for $4 per acre pays what per cent on the investment, if the average annual cost of fencing and taxes is 70 cents per acre?

8. Land valued at $80 per acre rents for $5 per acre. If the assessed value is one fifth the actual value and the taxes amount to $4\frac{1}{2}$% of the assessed value, and the average annual cost of fencing is 40 cents an acre, what per cent of the actual value does the land annually yield its owner?

"Marking Goods."

In many mercantile houses, the cost of goods is indicated by a private mark upon each article. The characters used may be ten letters, each letter standing for a figure. To aid the memory of those using the mark, the letters employed may be those of some "key word," as follows:

P u r c h a s e d
1 2 3 4 5 6 7 8 9

Any letter of the alphabet not in the key word may be used to represent *zero*. This makes it more difficult for a customer or a competitor to interpret the private mark than it would be, were a word of ten letters used, the tenth standing for *zero*. According to the above plan \$1.25 is *puh;* \$2.40 is *ucx* or *ucy* or *ucz;* \$2.05 is *ukh* or *ugh* or *ulh.*

In giving the answers to the following, write the *cost* in letters employing the key word given above; write the selling price in figures.

1. Men's rubber boots; list, \$4.20; discount, 33⅓%. (a) Cost? (b) Selling price to make a profit of 33⅓%? *

2. Since the discount in Problem 1 is 33⅓% and the expected profit 33⅓%, why is not the selling price the same as the list price?

3. Men's arctics; list, \$2.00; discount, 25 and 10%. (a) Cost? (b) Selling price to make a profit of 33⅓%?

4. Women's buckle gaiters; list, \$1.75; discount, 40%. (a) Cost? (b) Selling price to make a profit of 33⅓%?

5. Men's plow shoes; cost \$15 per case of 12 pairs. (a) Cost per pair? (b) Selling price to make a profit of 40%?

6. If a 36-pair case of Oxford Ties cost \$75.60, (a) what should be the cost mark on each pair, and (b) what the selling price, if the rate of profit is 33⅓%?

7. Shoe polish; list, \$10 per gross, actual cost 6¼¢ per bottle; selling price, 10¢ per bottle. (a) What was the per cent of discount? (b) What was the per cent of profit?

* In giving the selling price of articles worth one or more dollars each, make the right hand figure a *zero* or a *five*. To do this, mark above rather than below the given per cent of profit. For \$3.41 mark \$3.45; for \$2.17 mark \$2.20, etc.

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LATHS.

NOTE.—A *lath* is 4 feet long and $1\frac{1}{2}$ inches wide. Laths are laid about $\frac{3}{8}$ of an inch apart. Hence 1 bunch, 50 laths, will cover (including the spaces between them) a strip 4 feet wide and 50 times $1\frac{1}{2}+\frac{3}{8}$ inches long. Such a strip contains a little more than ——— square feet. Making large allowance for defective laths and wasted pieces, builders calculate that *a bunch of laths will cover 3 square yards.*

1. Estimate the laths necessary for two rooms, walls and ceilings, the rooms being 15 ft. by 16 ft. and 14 ft. by 18 ft. with ceilings 10 feet above the floors. Make no allowance for windows and doors in the making of this estimate. In practice, if the openings are numerous or large and "close figures" are required, such allowance may be made. In such cases it is best to make the computation as though there were no openings and make the proper deduction from the amount obtained.

2. Estimate the laths necessary for the walls of a 5-room house, the entire length of wall, including closets and halls, being 163 feet, and the height of wall being 8 feet, an allowance of 200 square feet to be made for openings.

3. Estimate the laths necessary for 6 ceilings the dimensions being as follows: 12 by $13\frac{1}{2}$, 12 by 14, 10 by $11\frac{1}{2}$, 9 by 13, $4\frac{1}{2}$ by 10, and 4 by 6.

SHINGLES.

NOTE.—*Shingles* are of variable widths; but in buying and selling and in making estimates, each 4 inches in width is counted as 1 shingle. A bunch of shingles is 20 inches wide; so there are 5 shingles in each course. There are 25 courses in each end of the bunch; hence there are ——— shingles in one end of the bunch and ——— shingles in the entire bunch. Four bunches are 1000 (1 M.) shingles. When shingles are laid "$4\frac{1}{2}$ inches to the weather," each shingle may be thought of as covering a space 4 inches by $4\frac{1}{2}$ inches, or 18 square inches. So 8 shingles are required for each square foot to be covered. To cover 100 square feet (1 square) 800 shingles will be required. But since there is always some waste and often short measure, builders

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calculate 900 shingles for each "square" (10 ft. by 10 ft.) to be covered. If the shingles are to be laid only 4 inches to the weather, 1 M. will be required for each square to be covered.

4. Estimate the shingles (to be laid 4 inches to the weather) for a double roof that is 36 feet long and whose rafters are 20 feet long. In giving the answer to this problem remember that lumber dealers will not divide a bunch of shingles.

5. Allowing 10% for waste, 1 thousand shingles laid 5 inches to the weather, will cover how many square feet?

6. Allowing 10% for waste and laying the shingles 5 inches to the weather, how many bunches must be purchased for a double roof that is 60 feet long and whose rafters are 26 feet long?

SIDING.

NOTE.—Siding is about $\frac{1}{4}$ inch in thickness and $5\frac{1}{2}$ inches wide, but it is counted as though it were an inch thick and 6 inches wide; that is, a 16-foot piece of siding is said to contain 8 feet of lumber. When siding is laid "4 inches to the weather," what per cent of itself must be added to the number of square feet to be covered to denote the number of feet of lumber required?

7. Estimate the siding for a building that is 30 feet by 40 feet and 18 feet to the plates,* the roof running lengthwise of the building and its highest point being 9 feet higher than the plates. Consider the openings (doors and windows) sufficient to make up for defective lumber and waste in cutting.

8. Allowing 7% for waste, 1 thousand feet of siding laid "4 inches to the weather" will cover how many square feet of surface?

9. A sufficient amount of siding was purchased to com-

*The *plates* are the horizontal timbers upon which the rafters rest near their lower ends. The height to the plates is practically the height to the gables. The gables are triangular, and in this case are 30 ft. by 9 ft.

plete a job, provided the siding was laid "4 inches to the weather." It was laid "$4\frac{1}{2}$ inches to the weather." What part of the amount purchased was unused?

MATCHED FLOORING.

NOTE.—Common matched flooring is, omitting the tongue, but little more than 5 inches in width, but since it is made from 6-inch boards, it is called 6-inch flooring and is counted as 1 foot of lumber for every 2 linear feet of flooring. To find the amount of flooring needed for a given surface *add $\frac{1}{6}$ of itself to the number of feet to be covered.*

10. Estimate the amount of 6-inch flooring needed for two floors throughout a building, main part, 24 by 36 with an ell 18 by 20.

11. Find the cost at $27 per M. of the flooring for the building described in Problem 10.

12. One thousand feet of 6-inch matched flooring will cover how many feet of surface?

SHEATHING.

NOTE.—A somewhat low grade of "matched flooring" is often used for the sheathing of buildings; that is, for a first covering of the frame work. The siding covers the sheathing.

13. Estimate the sheathing of "matched flooring" for a building 18 feet by 24 feet and 15 feet to the plates, the roof running lengthwise of the building and the highest point being 6 feet higher than the plates.

STOCK BOARDS.

NOTE.—Barns are sometimes covered with stock boards 1 foot in width. These boards are put on vertically and the cracks between them are covered with battens.

14. Estimate the stock boards for covering a barn that is 40 feet by 60 feet and 24 feet to the plates, the roof running lengthwise of the barn and its highest point being 10 feet higher than the plates. Consider the openings sufficient to

Estimates.

make up for the waste in cutting the lumber and for defective boards.

SIDEWALKS.

15. Find the cost of lumber for 240 feet of side walk 6 feet wide the walk to be made of 2-inch plank laid on three 2 by 6 stringers and the lumber to cost $17.50 per M.

16. Find the cost of lumber for 80 feet of walk, 3 feet wide, the walk to be made of 1-inch boards laid on two 2 by 4 stringers, the lumber to cost $16 per M.

BRICK AND MORTAR.

NOTE.—For data for estimating brick, see page 244 of this book.

17. Estimate the brick for the outer walls of a building, the outside perimeter of which is 240 feet, the height of the walls to the gables being 29 feet and the thickness, 3 bricks or, including the mortar, about 13 inches. Make no allowance for openings or "double corners" but assume that the material saved at these points will be sufficient to lay the gables.

18. To lay 1000 brick, about $1\frac{1}{8}$ bbl. lime and 13 bushels sand are required. Find the amount of lime and sand necessary to lay the brick in the outer walls of the building described in Problem 17.

19. Estimate the brick for the wall of a well 22 feet deep and 20 feet inside diameter, the brick to be laid without mortar and the wall to be 12 inches in thickness.*

STONE.

NOTE.—For data for estimating stone, see page 243 of this book.

20. Estimate the stone (cords) for the foundation of the outer walls of the building described in Problem 17, the

* For this estimate, it will be sufficiently accurate for practical purposes to use $3\frac{1}{7}$ as the ratio of diameter to circumference, to regard the length of the wall as equal to the circumference of a circle 21 feet in diameter, and to compute 25 bricks to the cubic foot.

foundation to be 5 feet high and 20 in. in thickness. Make no allowance for "double corners" or openings. The material saved at these points may be used in the expansion of the wall at the bottom.

21. Which is the cheaper, so far as material is concerned, a 12-inch (practically 13-inch) wall of brick or an 18-inch wall of stone, the brick costing $6.00 per M., and the stone $8.00 per cord?

PLASTERING.

22. Find the cost at 28¢ per square yard, of plastering three rooms, walls and ceilings, the dimensions of the rooms being as follows: Room 1, 18 ft. by 20 ft.; room 2, 14 ft. by 16 ft.; room 3, 13 ft. by 19 ft.; height of each room, 10 ft. Make no allowance for openings.

WATER.

NOTE.—In computing capacity for water, it is customary to regard 7½ gallons as equal to 1 cubic foot. Water weighs 62½ lb. to the cubic foot.

23. When the water in the well described in Problem 19, is 12 feet deep, how many gallons of water in the well?

24. A tank 10 feet deep and 12 feet in diameter will contain how many gallons of water?

25. What is the weight of the water that will exactly fill the tank described in Problem 24?

26. Estimate the weight of water sufficient to fill a tank as large as your school room, to the depth of 3 feet.

27. Estimate the number of gallons of water sufficient to cover the floor of your school room to the depth of 1 inch.

28. What is the weight of water to the acre when there is a rain-fall of 1 inch?

29. A heavy shower raised the water in a cistern 8 feet long and 6 feet wide, 2 feet 6 inches. The water came from

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a roof 30 feet long and 22 feet wide. The rain-fall was how many inches?

30. How many wells or cisterns 4 feet in diameter are equal in capacity to a well or cistern 20 feet in diameter, the depth being the same in each case?

31. Which would require the greater number of brick for walls, twenty-five 4-foot wells or one 20-foot well, the depth of the wells being the same in each case, but the 4-foot wells to have a 4-inch wall and the 20-foot well to have a 12-inch wall?

WEIGHT OF IRON.

NOTE.—Iron is about $7\frac{1}{4}$ times as heavy as water, the same bulk being considered.

32. (a) How much does a cubic foot of iron weigh? (b) A cubic inch?

33. Estimate the weight of a bar of iron, 1 inch by 6 inches by 12 feet.

34. (a) Estimate the weight of a 4-inch iron ball. (b) Of a 2-inch iron ball.

35. (a) Estimate the weight of an iron rod 1 inch in diameter and 12 feet long. (b) Of an iron rod 2 inches in diameter and 12 feet long.

36. Estimate the weight of a sheet of boiler iron, 8 feet square and $\frac{3}{8}$ of an inch thick.

37. Compare the weight of an iron ball 6 inches in diameter with the weight of one 2 inches in diameter.

WEIGHT OF ICE.

NOTE.—The specific gravity of a body that floats on water is less than 1; its weight is less than $62\frac{1}{2}$ lb. to the cubic foot. The fraction that indicates the part of a floating body that is below the surface of the water, also indicates its specific gravity. Thus, if three-fourths of a floating body is immersed, its specific gravity is $\frac{3}{4}$, and its weight is $\frac{3}{4}$ of $62\frac{1}{2}$ lb. to the cubic foot.

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38. Observe a piece of ice floating in water. Estimate (a) its specific gravity and (b) its weight per cubic foot.

39. Estimate the weight of a cake of ice 2 feet square and 18 inches thick.

40. Estimate the weight of ice (tons) that can be stored in an ice-house 20 feet square inside measurement, provided the depth of the ice is 12 feet and one foot of space next to the walls is filled with sawdust.

41. How many tons in an acre of ice that is 10 inches thick?

Weight of Lumber.

42. Observe a piece of partly seasoned pine scantling floating in water. Estimate (a) its specific gravity and (b) its weight per cubic foot.

43. Estimate the weight of a load of 12-foot pine fencing, the load being $3\frac{1}{2}$ feet wide and five feet high and the lumber "half-seasoned."

44. Estimate the weight of a pine log 16 feet long, 17 inches in diameter at one end and 19 inches at the other, the probable specific gravity being .8.*

Weight of Wood.

45. Observe a piece of oak or maple stove-wood floating in water. (a) Estimate its specific gravity; (b) estimate the weight of a cubic foot of it; and (c), on the supposition that a cord is equal to 100 solid feet of wood, estimate the weight of a cord of it.

46. The specific gravity of green hickory is almost 1. Estimate the weight of a round stick of it, 6 inches in diameter and 5 feet long.

* Although using the "average diameter," 18 in., will not give the exact solid content, it is sufficiently accurate for a practical estimate of weight. The fraction $\frac{3}{4}$ may be used as the approximate ratio of a circle to its circumscribed square.

FOOT-POUNDS.

NOTE.—A foot-pound is the amount of work required to lift one pound one foot.

47. How many foot-pounds of work must be done to raise 30 pounds 20 feet?

48. How many foot-pounds of work required to raise a man weighing 195 lb, to the third story of a building,—27 feet?

49. How many foot-pounds of work must a boy weighing 90 lb. do to raise himself to the second story of the school-house,—15 feet?

50. How many foot-pounds of work required, adding 25 % for friction and weight of ropes, chains, etc., to raise a stone weighing 340 lb. to the top of a monument 125 feet high?

51. How many foot-pounds of work required, adding 25 % for friction, etc., to raise a stone 2 ft. by 3 ft. by 18 in., specific gravity, $2\frac{1}{2}$, to the top of a wall 42 feet high?

52. How many foot-pounds of work required, adding 20 % for friction of machinery and of water in the pipes, to raise 5000 gallons of water 150 feet?

53. How many foot-pounds of work must a man do to carry himself, weight 150 lb., 18 bricks, weight 75 lb., and a hod, weight 12 lb., from the first floor to the third floor of a building,—22 feet?

54. How many foot-pounds of work required, adding 20 % for friction, weight of carrier, etc., to lift 18 bricks weighing 75 lb., 22 feet?

NOTE.—A horse-power is the force required to do 33000 foot-pounds of work per minute. The indicated horse-power is that exhibited at the engine doing the work. The effective or net horse-power is the indicated horse-power less losses by friction and imperfections of machinery. The effective horse-power is computed from the work actually done.

55. (a) Will more or less than 1 effective horse-power be required to lift 1 ton, 10 ft. high, in 1 minute? (b) To lift 1 ton, 20 ft. high, in 1 minute? (c) To lift 2 tons, 10 ft. high, in 1 minute? (d) To lift 2 tons, 20 ft. high, in 2 minutes? (e) To lift 50 gallons of water, 100 feet high, in 1 minute? (f) To lift 8 cubic feet of water, 60 feet high, in 1 minute?

56. How many effective horse-power required to lift 5000 gallons of water, 100 feet, in 1 hour?

57. How many indicated horse-power required to do the work described in Problem 56, provided 30 % of the force is lost by friction and imperfections of machinery?

58. How many indicated horse-power required to raise 2000 cubic feet of water, 80 feet, in 1 hour, provided 30 % of the power is lost by friction of machinery and of water in the pipes?

59. If a horse pushes into the collar continuously with a force of 150 lb. and travels at the rate of $2\frac{1}{2}$ miles an hour, (a) how many foot-pounds per minute does his work represent? * (b) Is this more or less than 1 horse-power?

60. If 30% of the force of an engine is lost in friction and in the imperfections of the machinery, how many gallons of water per hour can be raised 150 feet by a 20 horse-power engine?

61. What is the effective horse-power of an engine that can raise 1500 pounds 2376 feet in 6 minutes?

62. What is the effective horse-power of an engine that can raise 150 cubic feet of water per minute from a mine 264 feet deep?

* Pushing continuously into the collar with a force of 150 lb. and moving $2\frac{1}{2}$ miles an hour while doing this, is equivalent to lifting 150 lb. as many feet in an hour, as there are feet in $2\frac{1}{2}$ miles.

NOTE.—The remaining pages of this book are devoted to problems that are distinctively agricultural. They may be omitted by pupils and teachers who are in no way interested in such work.

The efforts that are being made in many states to introduce the elements of agricultural science into the rural schools, the thousands of boys in the public schools who will soon be, either directly or indirectly, interested in such problems as are here presented, the great and increasing interest in the rural school problem, and the fact that the teachers for the rural schools are, for the most part, educated in the city schools, is the apology, if one is needed, for introducing this topic in the pages of a common school arithmetic.

The problems in *nutritive ratio* on pages 316 and 317, are as thoroughly "practical" as any in the book. The proper "balancing" of foods with reference to their muscle-forming and their fat-forming elements, is receiving much attention in the Agricultural Colleges and Experiment Stations as well as among the more intelligent stock feeders everywhere. Enterprising young farmers in all sections of the United States, are asking for information in regard to the method of computing the "*balanced ration.*" Hence a few simple problems under this head are introduced, and the author confidently believes that the skillful teacher will so use these pages as to make them of great value in aiding a large class of boys to see the relation of their school work to their prospective life work.

The Comparative Value of Land for Agricultural Purposes.

1. If a certain piece of land, with an average annual expenditure of $17 per acre, gives an average annual return of $20 per acre, and is therefore considered worth $40 an acre as an investment, what is that piece of land worth that, with the same average expenditure, will give an average annual return of $29 per acre?

2. One piece of land produces 40 bushels of corn to the acre; another piece produces 60 bushels. If the cost of producing and marketing is $8 per acre in each case, and if the average price of corn is 25 cents a bushel, and if the difference in yield is due to the nature of the soils and not to differences in methods of cultivation, what is the relative value of the two pieces of land as an investment?

Shrinkage of Corn.

At the Agricultural Experiment Station, Ames, Iowa, in the autumn of 1898, a corn-crib was built upon the platform of a set of farm scales. On Oct. 19, 7000 lb. of corn was husked and stored in the crib. The corn was weighed once a week for a year with the following results:

Oct. 19, 7000	Feb. 15, 6290	June 14, 5860
Oct. 26, 6835	Feb. 21, 6250	June 21, 5880
		June 28, 5840
Nov. 2, 6715	Mar. 1, 6245	
Nov. 9, 6475	Mar. 8, 6225	July 5, 5825
Nov. 16, 6430	Mar. 15, 6310	July 12, 5780
Nov. 23, 6540	Mar. 22, 6280	July 19, 5760
Nov. 30, 6425	Mar. 29, 6235	July 26, 5700
Dec. 7, 6410	Apr. 5, 6220	Aug. 2, 5730
Dec. 14, 6380	Apr. 12, 6100	Aug. 9, 5800
Dec. 21, 6380	Apr. 19, 5980	Aug. 16, 5740
Dec. 28, 6375	Apr. 26, 5950	Aug. 30, 5700
Jan. 4, 6375	May 3, 5970	Sept. 6, 5670
Jan. 11, 6370	May 10, 5940	Sept. 13, 5640
Jan. 18, 6370	May 17, 5950	Sept. 20, 5610
Jan. 25, 6360	May 24, 5925	Sept. 27, 5630
	May 31, 5900	
Feb. 1, 6330		Oct. 4, 5570
Feb. 8, 6310	June 7, 5860	Oct. 19, 5620

1. Find the per cent of shrinkage from Oct. 19 to Oct. 19 of the next year.

2. *Using 7000 lb. as the base*, find the per cent of shrinkage for each monthly period,—Oct. 19 to Nov. 2; Nov. 2 to Dec. 7; Dec. 7 to Jan. 4; Jan. 4 to Feb. 1, etc.

3. Find the sum of the thirteen answers to Problem 2 and compare this with the answer to Problem 1.

Shrinkage of Corn and Hay.

1. Making no allowance for loss by rats and mice, for risk of destruction by fire or tornado, or for extra labor in handling, what per cent above the Oct. 19th price, should a farmer receive for corn on Jan. 4th to make up for the loss by shrinkage as shown in the table on the preceding page?

2. What per cent above the Jan. 4th price, should a farmer receive for corn on the following Sept. 6th to make up for loss by shrinkage as shown in the table?

3. If new corn is 40 cents per bushel of 75 lb., Nov. 2nd, what should be the price per bushel of 70 lb., on the following Aug. 2nd to make up for shrinkage as shown on the preceding page?

4. Which is the better for the seller, corn at 50 cents per cwt. from the field Oct. 19th, or corn at 41 cents a bushel from the crib on the following June 7th?

5. Which is the better for the buyer, new corn at 37½ cents for 75 lb. on Nov. 2nd, or the same corn on the following May 3rd at 37½ cents for 70 lb.?

6. Making proper allowance for the use of money, for risk of loss by vermin, fire, and tornado, for extra labor, and for shrinkage, what, in your judgment,* should the farmer receive for corn 70 lb. to the bushel, on July 5th, to be equivalent to 75 lb. for 30 cents, drawn directly from the field Oct. 19th?

7. If clover hay shrinks in weight 30% from June to October, what should be the price in October to correspond with $6.00 per ton in June?

8. If timothy hay shrinks in weight 20% from July to October, what should be the price in October to correspond with $7.00 per ton in July?

*The author very well knows that many pupils will be confronted with this problem who can have no "judgment" regarding it. Such may omit it. For those who can understand the conditions, the problem may prove the most valuable of any on the page.

At the Slaughter-House.

Shrinkage in dressing.—Beeves shrink from 43 to 55% in dressing. This means that a carcass of dressed beef weighs from 43 to 55% less than the live weight of the animal. Hogs shrink from 15 to 22%.

BEEVES.

No.	Live Weight.	Cost per cwt.	Dressed Weight.	Weight of Hide.
1.....	1740	$5.20	990	105
2.....	1230	4.30	660	65
3.....	1450	4.75	750	80
4.....	1570	5.00	810	85
5.....	1080	4.10	590	48
6.....	1190	4.60	610	52
7.....	1240	4.40	690	60
8.....	1630	5.00	830	87
9.....	1470	4.80	820	63
10.....	1360	4.90	710	56

(a) Find the per cent of shrinkage in each of the above cases.

(b) If the tongue, liver, heart, and offal are worth enough to pay for slaughtering, and the hide is worth 7 cents per pound, what is the cost per cwt. of each carcass of beef?

(c) Find the per cent of shrinkage on the entire lot of ten head.

(d) Find the average cost of the dressed beef per cwt.

HOGS.

11. Live weight, 450 lb.; dressed weight, 370 lb.
12. Live weight, 650 lb.; dressed weight, 540 lb.
13. Live weight, 260 lb.; dressed weight, 220 lb.
14. Live weight, 350 lb.; dressed weight, 290 lb.
15. Live weight, 340 lb.; dressed weight, 270 lb.

(e) Find the per cent of shrinkage in each of the above cases.

(f) If live hogs are worth $4.80 per cwt., and the cost of slaughtering is 50 cents per head, what is the cost per cwt. of each of the above carcasses of pork?

An Experiment in Feeding Calves.

Twelve calves were divided into three lots of four each and fed for 74 days. The following table shows the kind and amount of food consumed and the gain of the calves in weight:

Feed, etc.	Lot I.	Lot II.	Lot III.
Skim-milk	3572 lb.	3760 lb.	3759 lb.
Hay	1481 "	1478 "	1484 "
Oat meal	605 "		
Corn meal		538 "	601 "
Flax seed		59 "	
Gain in weight	498 "	489 "	509 "
Average daily gain per head	a	b	c
Cost of food per pound of gain	d	e	f

1. Find the average daily gain per head, and the cost of food per pound of gain, the following being the value per cwt. of each kind of food: Skim-milk, 12c.; hay, 30c.; oat meal, 75c.; corn meal, 65c.; flaxseed, $2.50.

Experiments in Feeding Swine.*

The following gives the averages of many hundred experiments in feeding swine. "Pounds of feed" means pounds of grain *or its equivalent.* Six pounds of skim-milk or 12 pounds of whey were counted as 1 " pound of feed." This is in accord with the Danish valuation.

Weight of pigs in pounds.	Pounds of feed per head per day.	Pounds of gain per head per day.	Feed required for 100 pounds gain.
15 to 50	2.23	.76	a
50 to 100	3.35	.83	b
100 to 150	4.79	1.10	c
150 to 200	5.91	1.24	d
200 to 250	6 57	1.33	e
250 to 300	7.40	1.46	f
300 to 350	7.50	1.40	g

2. Find the number of pounds of feed required for 100 lb. of gain in each instance.

*The data given under this head are from FEEDS AND FEEDING by Prof. W. A. Henry of the University of Wisconsin. The tests were made at Agricultural Experiment Stations in many states.

"Nutritive Ratio."

The chemist sometimes divides the dry part of food constituents into three groups, namely: protein compounds, carbohydrates, and ether-extract.*

Protein contains nitrogen and is a "muscle former."

The carbohydrates contain no nitrogen and are "fat formers."

Ether-extract contains no nitrogen and is a "fat former."

The nutritive ratio is the ratio of the muscle formers to the fat formers in any article of food. Since ether-extract is more valuable as a food constituent than the carbohydrates, it is customary to multiply the digestible ether-extract by 2.4 before making the comparison. Thus; the digestible constituents of 100 lb. of clover hay are as follows: Protein, 6.8 lb.; carbohydrates, 35.8 lb.; ether-extract, 1.7 lb. In determining the nutritive ratio of clover hay, 6.8 lb. is compared with 35.8 lb. + 2.4 times. 1.7 lb., or 6.8 : 39.88 = 1 : 5.8+.†

Food.‡	Digestible substance in 100 pounds.			Nutritive ratio.
	Protein.	Carbohydrates.	Ether-Extract.	
Corn stover §	1.7	32.4	0.7	a
Red clover hay	6.8	35.8	1.7	b
Timothy hay	2.8	43.4	1.5	c
Oat straw	1.2	38.6	0.8	d
Corn (shelled)	7.8	66.7	4.3	e
Oats (grain)	9.2	47.3	4.2	f
Wheat bran	12.2	39.2	2.7	g
Oil meal	29.3	32.7	7.0	h
Skim-milk	2.9	5.2	0.3	i
Butter-milk	3.9	4.0	1.1	j

1. Find the nutritive ratio for each kind of food mentioned in the table.†

* The protein compounds, or proteids, include gluten, albumin, casein, etc. The carbohydrates consist of carbon, oxygen, and hydrogen, and include sugar, starch, and cellulose. The term, ether-extract is applied to a variety of compounds the most important of which is oil or "fat."

† This ratio is usually given with a unit for the first term and with the second term true to tenths.

‡ The figures in this table are taken by permission from FEEDS AND FEEDING by Prof. W. A. Henry of the University of Wisconsin.

§ "The dry stocks of corn from which the ears have been removed." HENRY.

The "Balanced Ration."

1. What is the nutritive ratio of a ration consisting of equal parts of clover hay and oat straw?

2. What is the nutritive ratio of a feed of corn and skim-milk, mixed in the proportion of 2 lb. of milk to 1 lb. of corn?

3. The theoretical nutritive ratio of food adapted to the needs of a horse engaged in light work, is 1 : 7. Find the nutritive ratio of each combination of foods named below and note whether it is *wider* or *narrower** than 1 : 7.

(a) Two parts by weight of timothy hay to one of corn.

(b) Two parts clover hay to one part of oats.

(c) Five parts clover hay, five parts timothy hay, five parts corn, and one part oil meal.

4. The theoretical nutritive ratio of food adapted to fattening swine, first period,† is 1 : 5.9. Find the nutritive ratio of each combination of foods named below, and note whether it is *wider* or *narrower** than 1 : 5.9.

(a) One part by weight of corn to one part of oats.

(b) Ten parts corn, ten parts skim-milk, and one part oil meal.

5. The theoretical nutritive ratio of food adapted to the needs of a milch cow weighing 1000 lb. and giving daily 22 lb. (about 11 quarts) of milk, is 1 : 5.7. Find the nutritive ratio of each combination of foods named below, and note whether it is *wider* or *narrower** than 1 : 5.7.

(a) Three parts timothy hay and one part of corn.

(b) Ten parts clover hay, ten parts corn stover, five parts corn, five parts wheat bran, and two parts oil meal.

*A nutritive ratio in which the proportion of fat-formers to muscle-formers is large (as, oat straw, 1:33.7) is said to be a *wide ratio;* while one in which this proportion is comparatively small (as oil meal, 1 : 1.7) is said to be a *narrow* nutritive ratio. The nutritive ratio of corn (1 : 9.8) is *wider* than that of oats (1 : 6.2).

† Writers upon this subject divide the fattening time into three periods. The proper nutritive ratio is *wider* for the second period than for the first; and *wider* for the third period than for the second.

On the Farm.

1. If hills of corn are $3\frac{3}{5}$ ft. apart each way, how many hills to the acre?

2. If, in a field of corn planted as stated in Problem 1, there is on the average $1\frac{1}{2}$ lb. of corn to each hill, counting 80 lb. to the bushel to allow for shrinkage, what is the yield per acre?

3. If potatoes are planted in rows 3 feet apart, and in hills 2 feet apart in the row, how many hills to the acre?

4. If, in a field planted as stated in Problem 3, the yield on the average is $1\frac{1}{2}$ lb. to the hill, what is the yield per acre?

5. How many rods of fence will enclose a 40-acre square field and divide it into 10-acre square fields?

6. If the posts are placed exactly 1 rod apart, how many will be required for the fencing described in Problem 5?

7. How far must a man travel in plowing 3 acres of land (a) with a 16-inch plow? (b) With a gang of 2 12-inch plows?

8. How many 160-acre farms in a township 6 miles long and 6 miles wide?

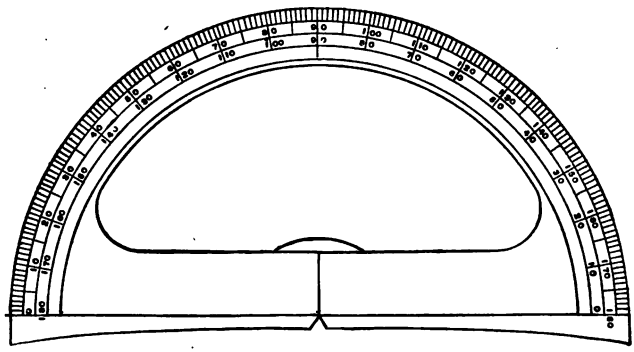
9. If there is a 4-rod road on every section line* of a township exactly 6 miles square, (a) how many acres of the township in its roads? (b) How many acres of each square 160-acre farm are taken for roads?

10. If in mowing a square 10-acre field, one begins on the outer edge of the field and mows around it thus working towards the center, what part of the field is mowed when a strip 4 rods wide has been cut?

11. If corn is planted as stated in Problem 1, and when mature is cut and put into shocks each shock containing the corn from a piece 12 hills square, how many shocks to the acre?

*A section is 1 mile square, and half of the width of the road is on each side of every section line.

PROTRACTOR.



Carefully paste this sheet upon card-board; then cut out the protractor with a sharp knife and preserve it for use in measuring and in constructing angles. See pages 99, 109, 119, etc.